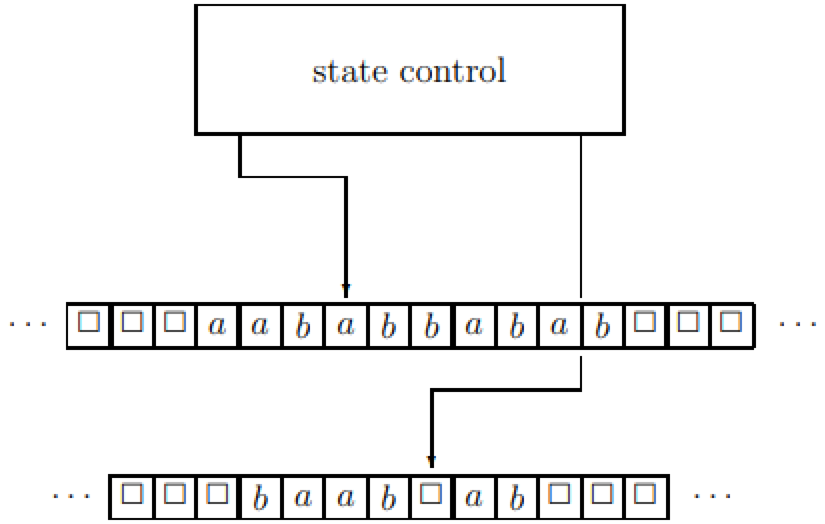
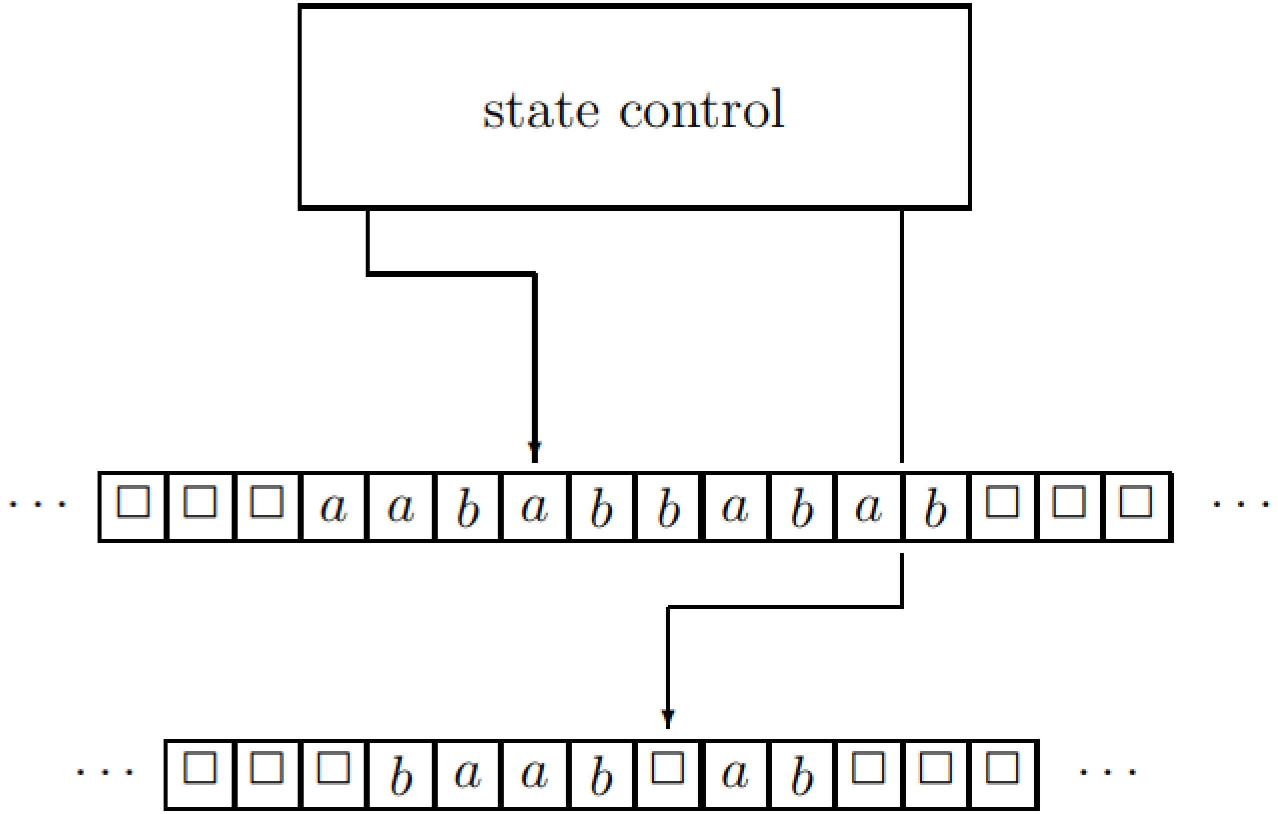


Theory of Computation

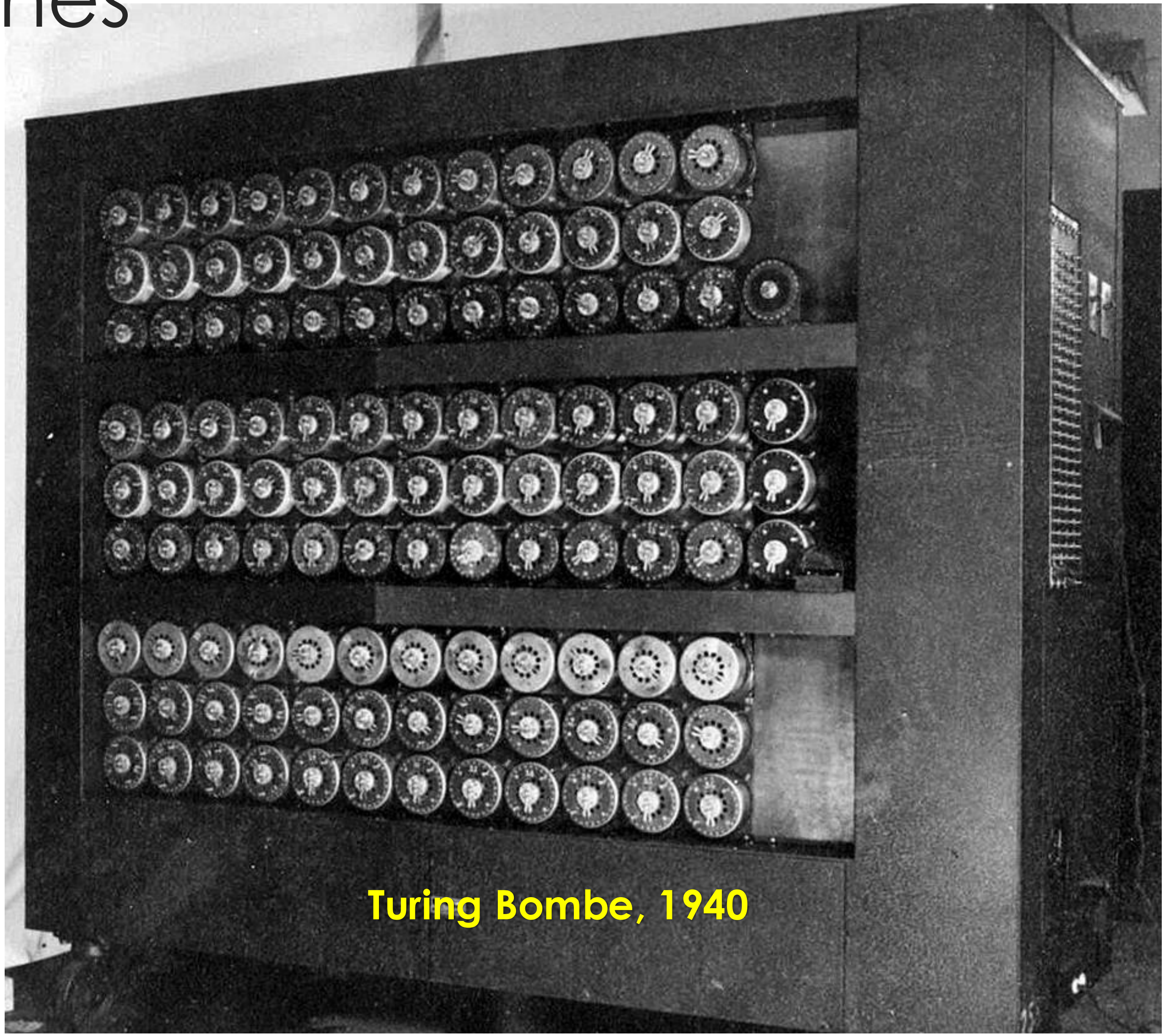
Lesson 10

Turing Machines

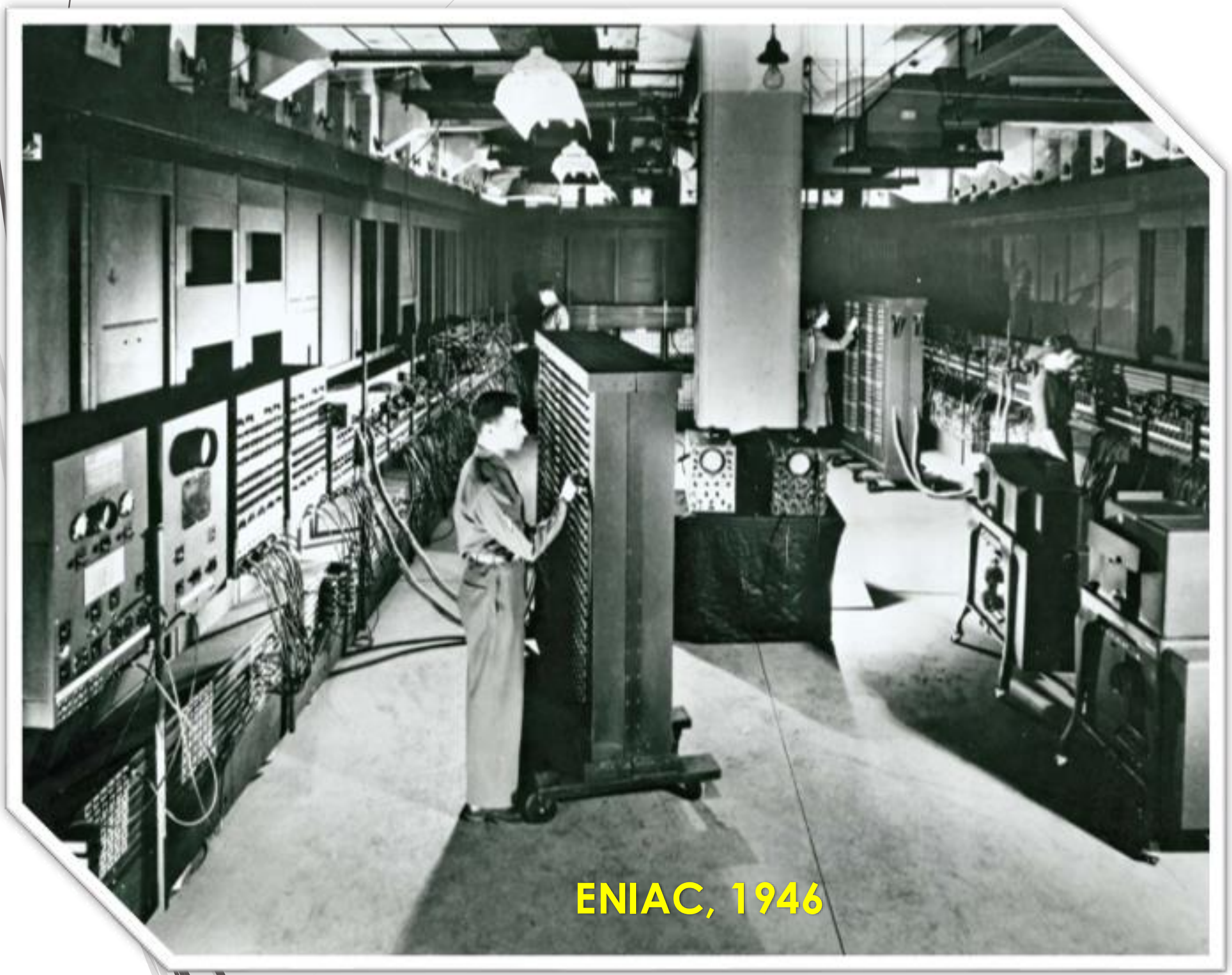
Turing Machines

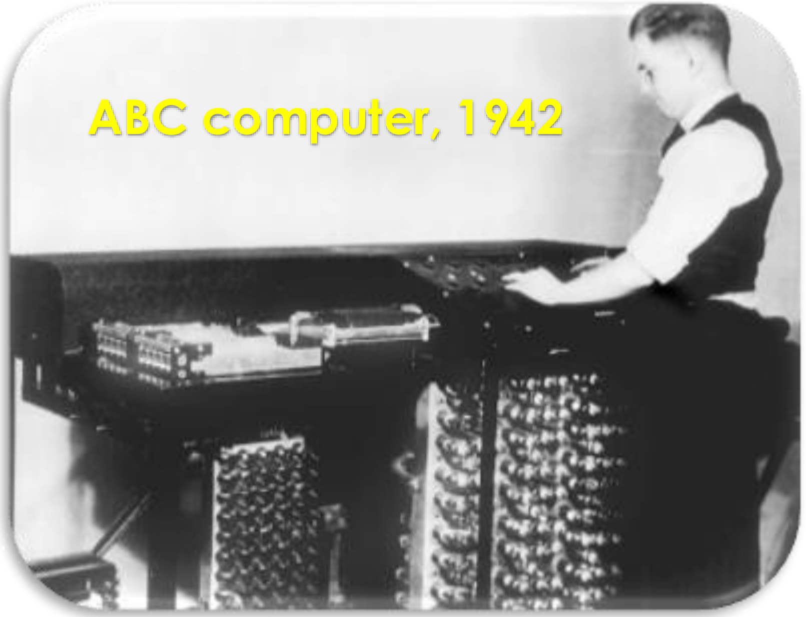


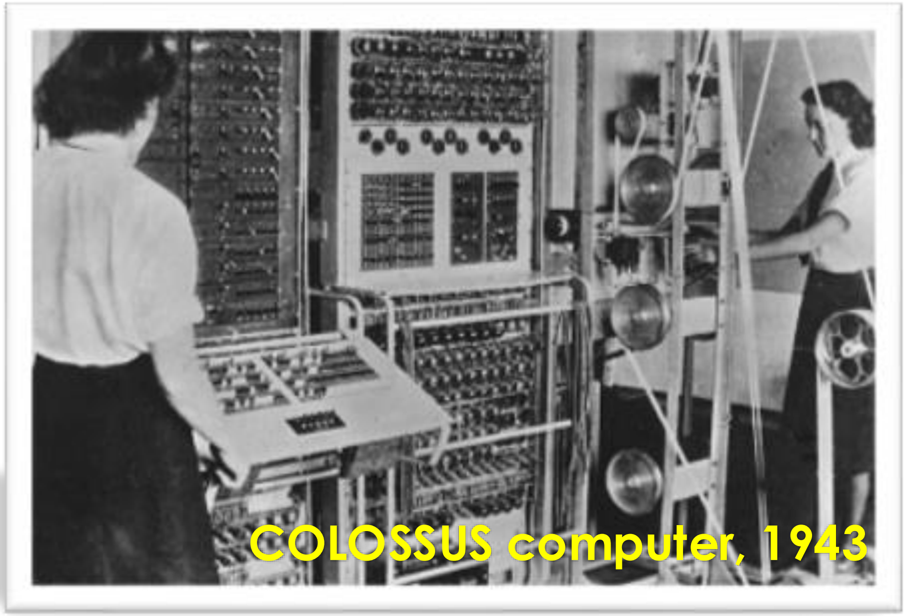
Turing Machines



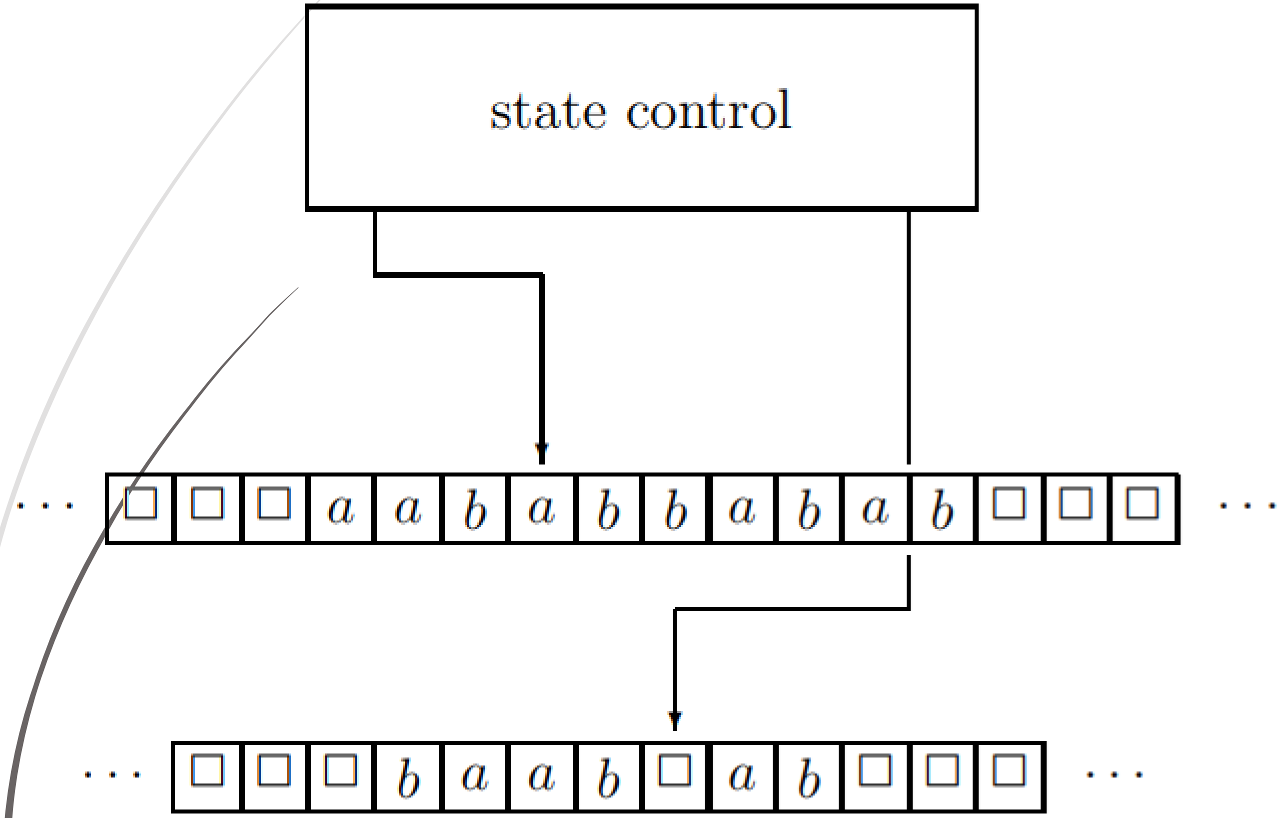
Turing Machines





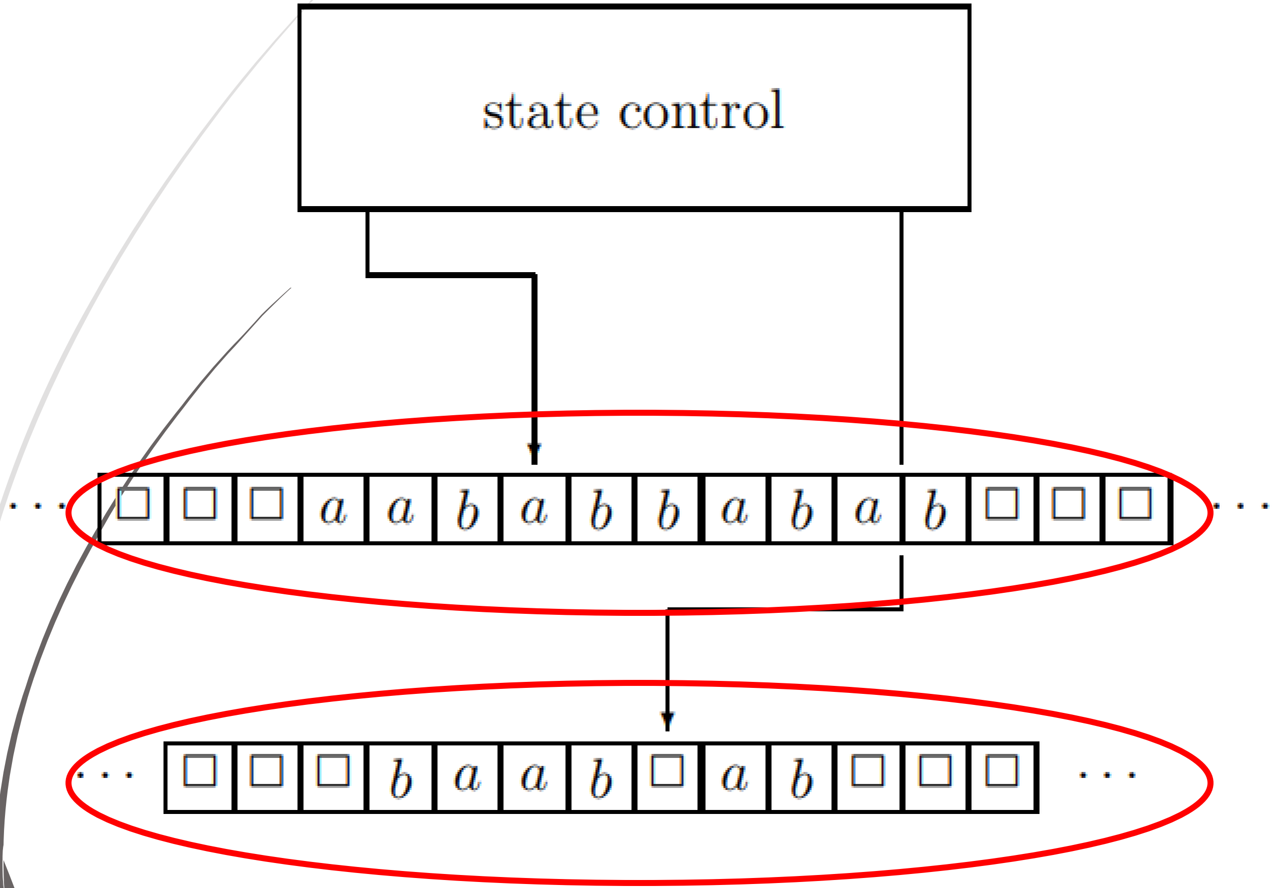


Turing Machines



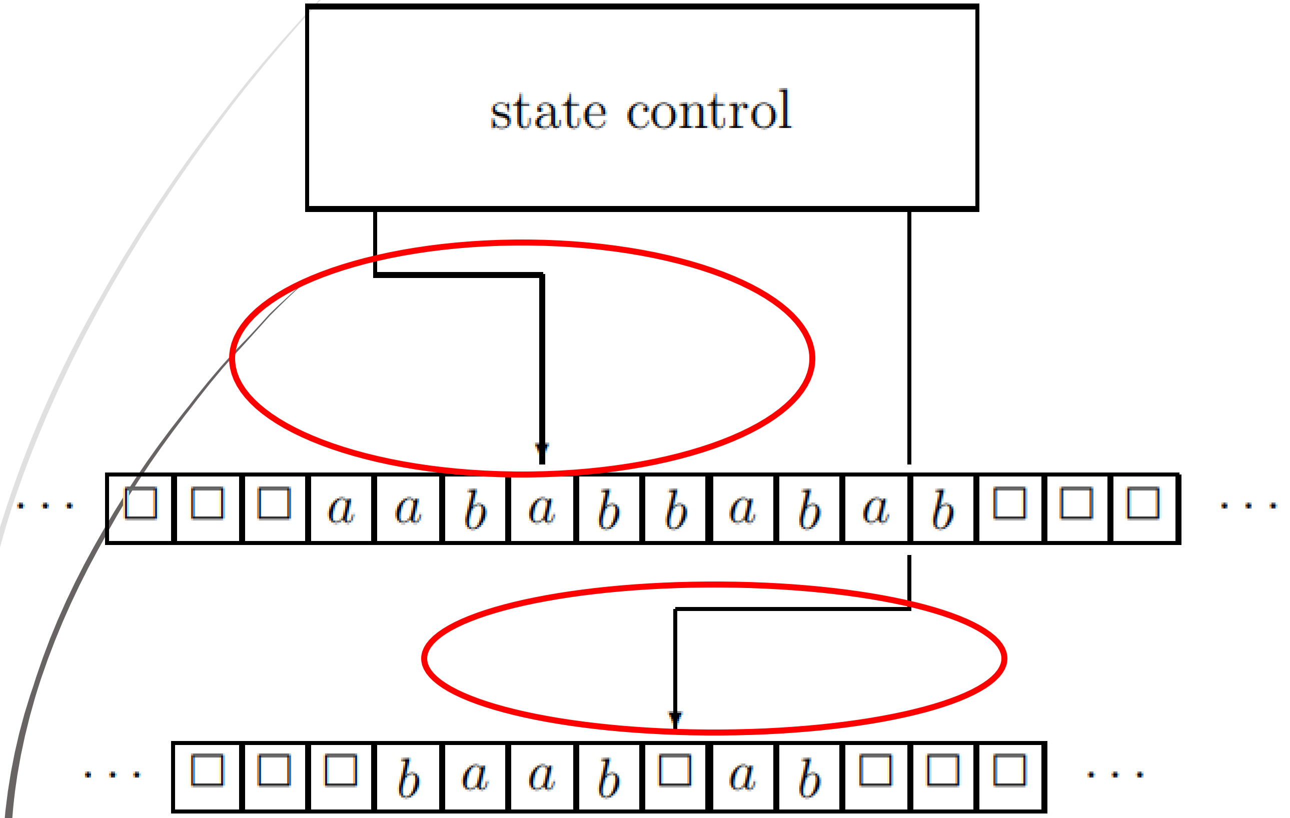
Although Turing machines are similar to finite automata, because of infinite length writable tape, have the advantage of unlimited memory.

Turing Machines



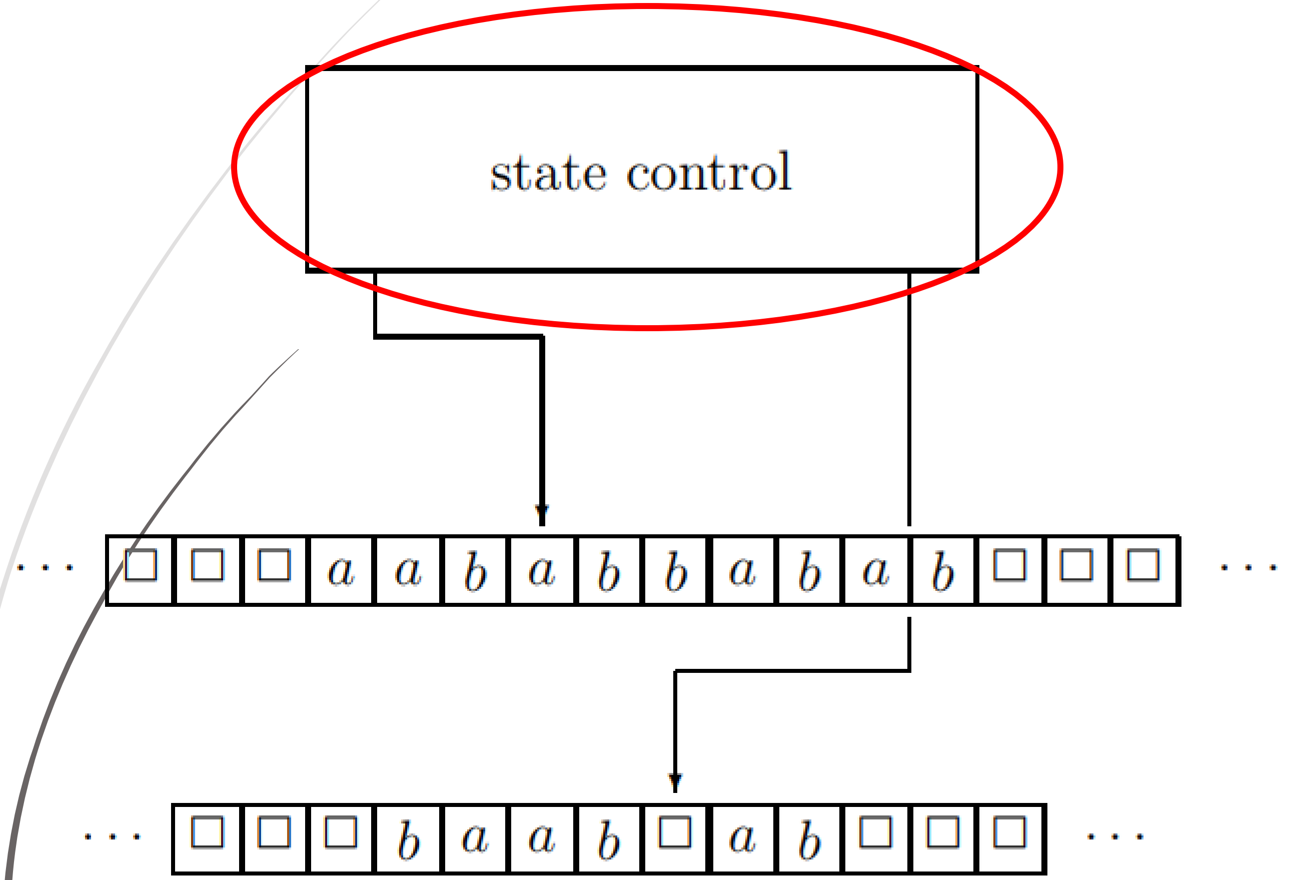
1. There are k tapes, k ≥ 1. Each tape has infinite cells. Each cell stores a symbol belonging to a finite set Γ. If a cell contains #, then this means that the cell is actually empty.

Turing Machines



2. Each tape has a tape head which can move along the tape, one cell per move. It can also read the cell it currently scans and replace the symbol in this cell by another symbol.

Turing Machines



3. There is a state control, which can be in any one of a finite number of states. The finite set of states is denoted by Q. The set Q contains three special states: a start state, an accept state, and a reject state.

Formal Definition

A deterministic Turing machine is a 7-tuple

$M = (\Sigma, \Gamma, Q, \delta, q, q_{accept}, q_{reject})$, where

- Σ is a finite set, called the input alphabet; the blank symbol # is not contained in Σ,
- Γ is a finite set, called the tape alphabet; this alphabet contains the blank symbol #,
- Q is a finite set, whose elements are called states,
- q, q_{accept}, q_{reject} are the start, accept, reject states.
- δ is called the transition function, which is a function

$$Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, N, R\}^k$$

A Sample Delta Rule

$qa \rightarrow q\#R$

states

When we focus on the states here, we will see that the state has not changed. Thus, we can read it as 'stay in the same state'.

A Sample Delta Rule

$qa \rightarrow q\#R$

tape

When the tape head reads a symbol ‘a’ from the tape, the tape head must delete it.

A Sample Delta Rule

$qa \rightarrow q\#R$

tape head action

On the right hand-side, there is a capital R symbol. We can understand that 'tape head must move to the right'.

A Sample Delta Rule

$qab \rightarrow q\#bRN$

When the state controller is in the state q, if the first tape head reads a symbol a and the second tape head shows a symbol b,

A Sample Delta Rule

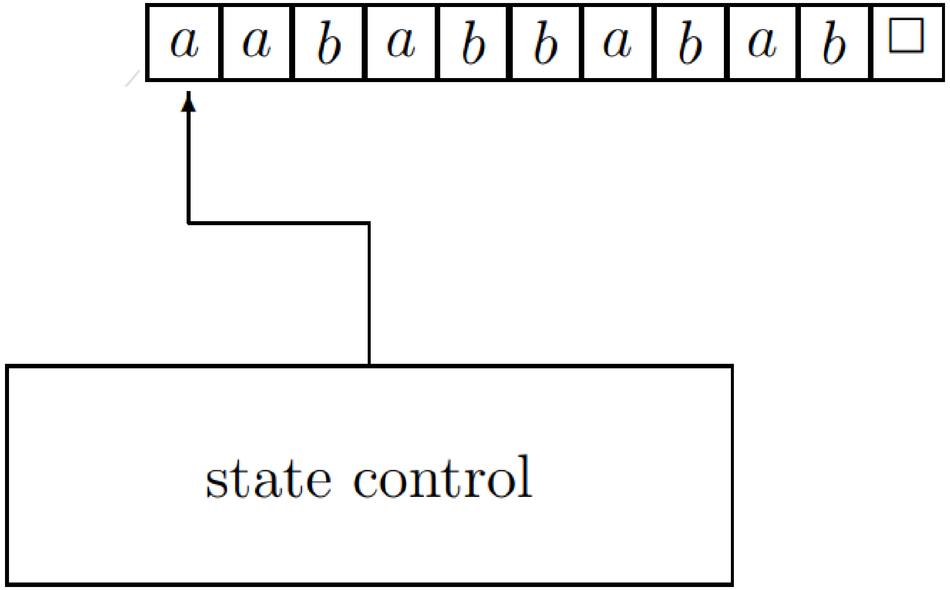
$qab \rightarrow$ q#bRN

Then stay in the same state,

In the first tape, delete the symbol by moving right,

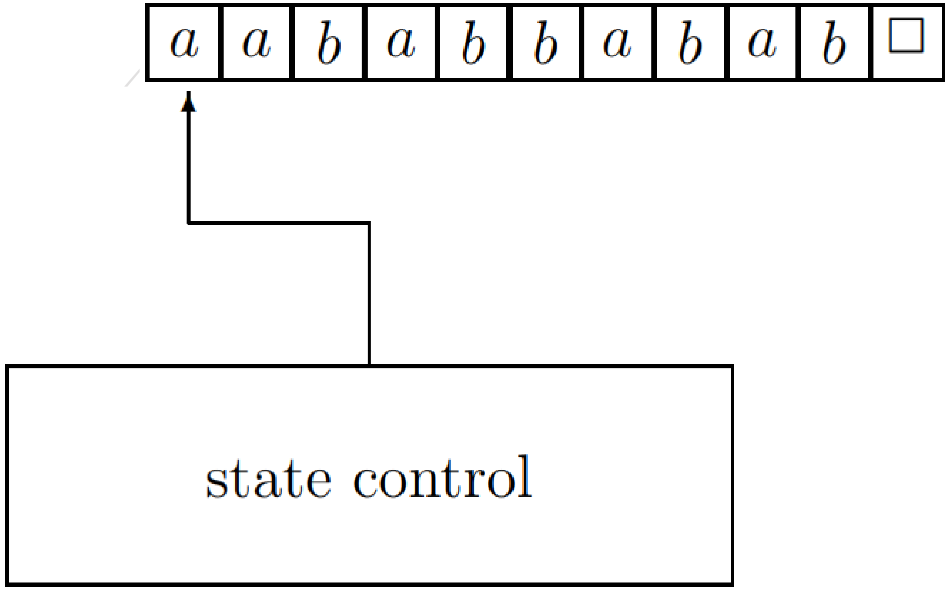
In the second tape, do not change tape symbol and not move.

Start Configuration of TM



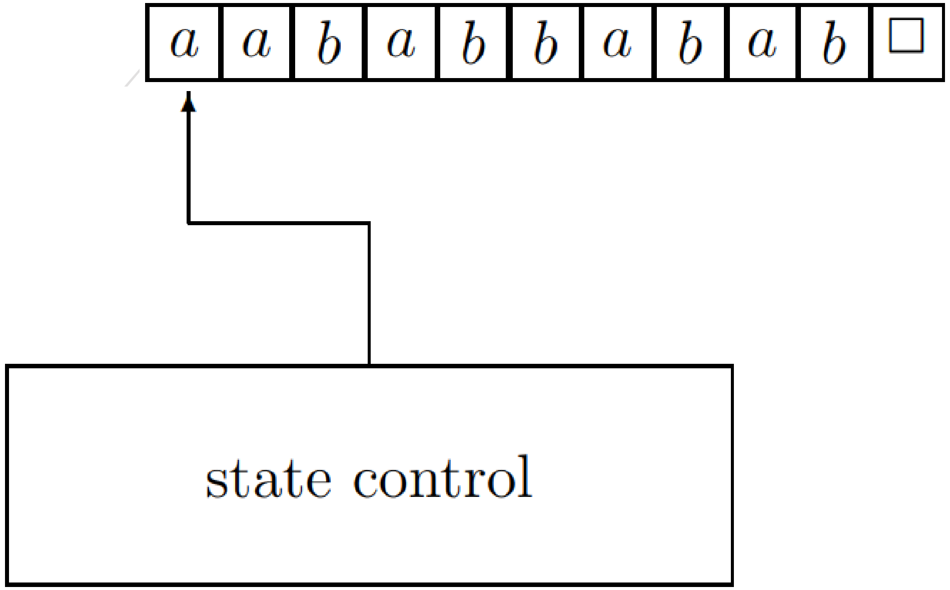
The input is a string over the input alphabet Σ. Initially, this input string is stored on the first tape, and the head of this tape is on the leftmost symbol of the input string. Initially, all other k – 1 tapes are empty, i.e., only contain blank symbols, and the Turing machine is in the start state q.

Execution of TM



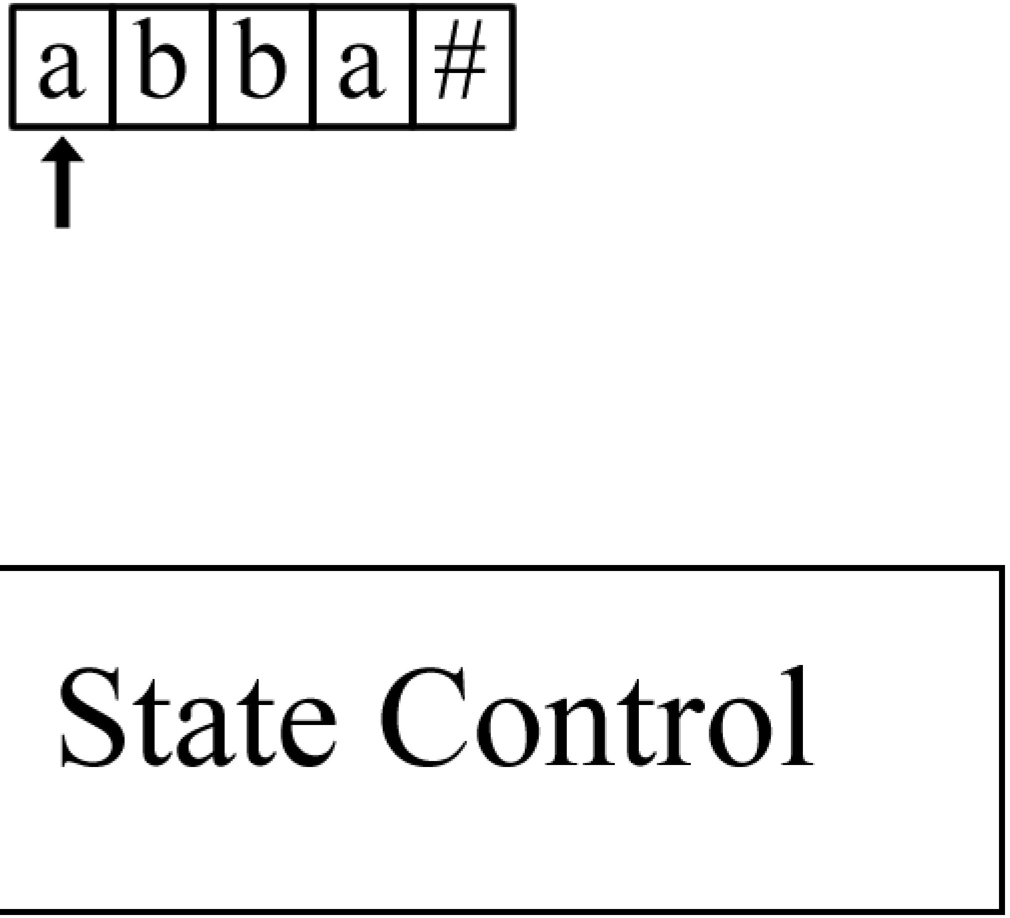
Starting in the start configuration, the Turing machine performs a sequence of computation steps as described before. The computation terminates at the moment when the Turing machine enters q_{accept} or q_{reject}.

Acceptance of TM



If the computation on the input string terminates in q_{accept}, the Turing machine M accepts this input. If the computation on this input terminates in q_{reject}, then M rejects the input string.

Example 1: Palindromes using one tape



Example 1: Palindromes using one tape

$$q_0 a \rightarrow q_a \square R$$
$$q_0 b \rightarrow q_b \square R$$
$$q_0 \square \rightarrow q_{accept}$$

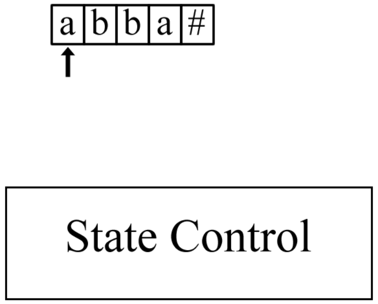
$$q_a a \rightarrow q_a a R$$
$$q_a b \rightarrow q_a b R$$
$$q_a \square \rightarrow q'_a \square L$$

$$q_b a \rightarrow q_b a R$$
$$q_b b \rightarrow q_b b R$$
$$q_b \square \rightarrow q'_b \square L$$

$$q'_a a \rightarrow q_L \square L$$
$$q'_a b \rightarrow q_{reject}$$
$$q'_a \square \rightarrow q_{accept}$$

$$q'_b a \rightarrow q_{reject}$$
$$q'_b b \rightarrow q_L \square L$$
$$q'_b \square \rightarrow q_{accept}$$

$$q_L a \rightarrow q_L a L$$
$$q_L b \rightarrow q_L b L$$
$$q_L \square \rightarrow q_0 \square R$$



Example 2: Palindromes using two tapes

$$q_0 a\Box \rightarrow q_0 aaRR$$
$$q_0 b\Box \rightarrow q_0 bbRR$$
$$q_0 \Box\Box \rightarrow q_1 \Box\Box LL$$

$$q_2 aa \rightarrow q_2 aaRL$$
$$q_2 ab \rightarrow q_{reject}$$
$$q_2 ba \rightarrow q_{reject}$$
$$q_2 bb \rightarrow q_2 bbRL$$
$$q_2 \Box\Box \rightarrow q_{accept}$$

$$q_1 aa \rightarrow q_1 aaLN$$
$$q_1 ab \rightarrow q_1 abLN$$
$$q_1 ba \rightarrow q_1 baLN$$
$$q_1 bb \rightarrow q_1 bbLN$$
$$q_1 \Box a \rightarrow q_2 \Box aRN$$
$$q_1 \Box b \rightarrow q_2 \Box bRN$$
$$q_1 \Box\Box \rightarrow q_{accept}$$

- That’s all.
- Thanks for listening.