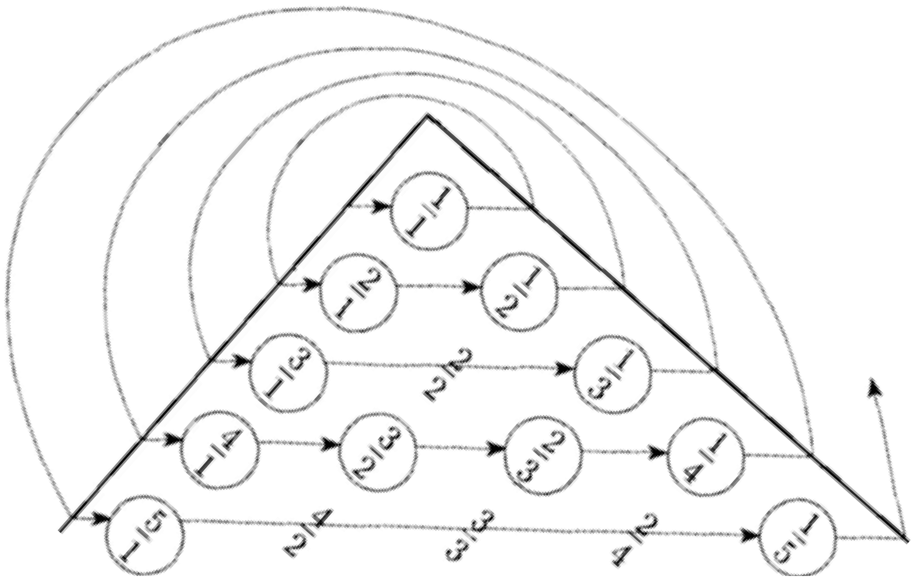


Theory of Computation

Countable & Uncountable Languages

Cardinality

Cardinality is simply a measure of the number of elements of a set. For example, the set A={a, b, ab, ba} contains 4 elements, and therefore A has a cardinality of 4.

- *The size or Cardinality of A is 4*
- $|A|=4$

But what about those with infinite elements? For example, what is the cardinality of a language-L made up of binary words?

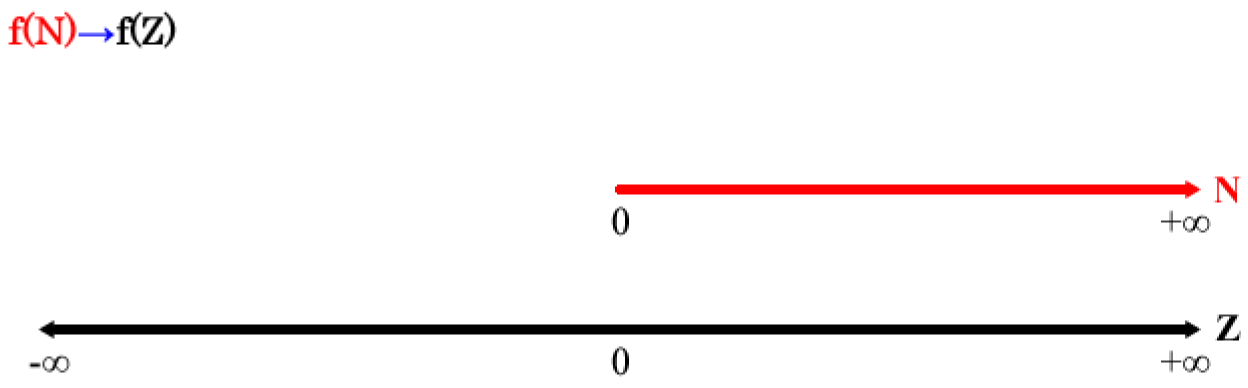
Countable Sets

- ✓ Any set X with cardinality less than that of the natural numbers is said to be a **countable** (finite) set.
- ✓ Any set X that has the same cardinality as the set of the natural numbers is said to be a **countable** (countably infinite) set.
- ✓ Any set X with cardinality greater than that of the natural numbers is said to be **uncountable**.

We will focus on counting using **Natural numbers**.

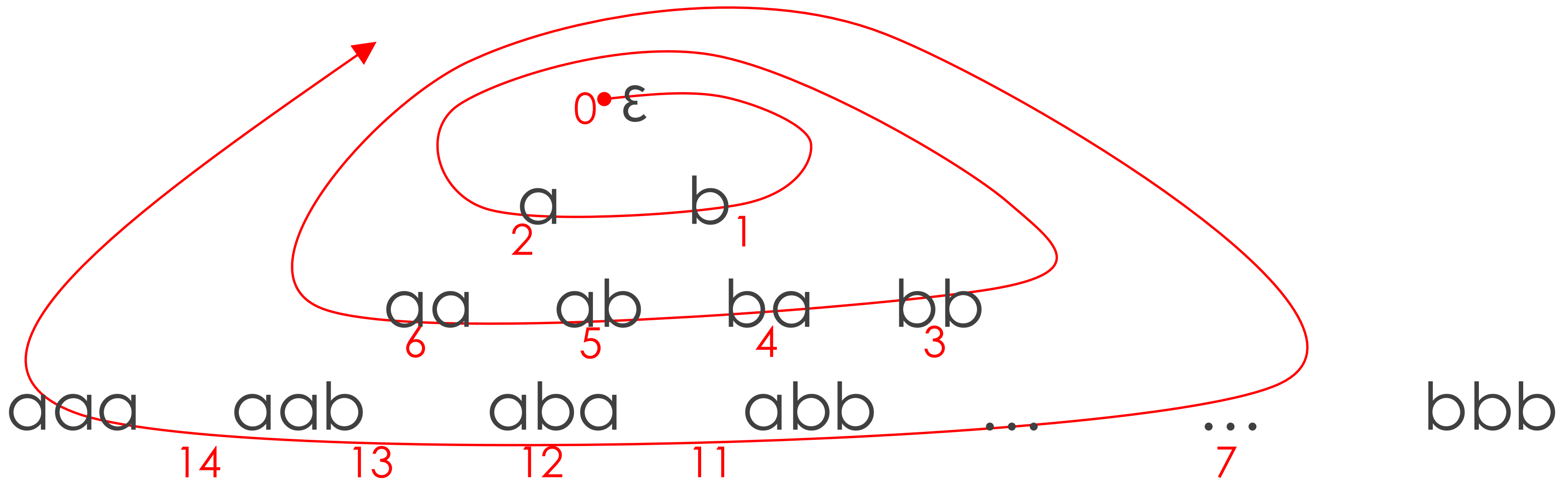
Countable Sets

✓ The set of integer numbers is **Countable**.



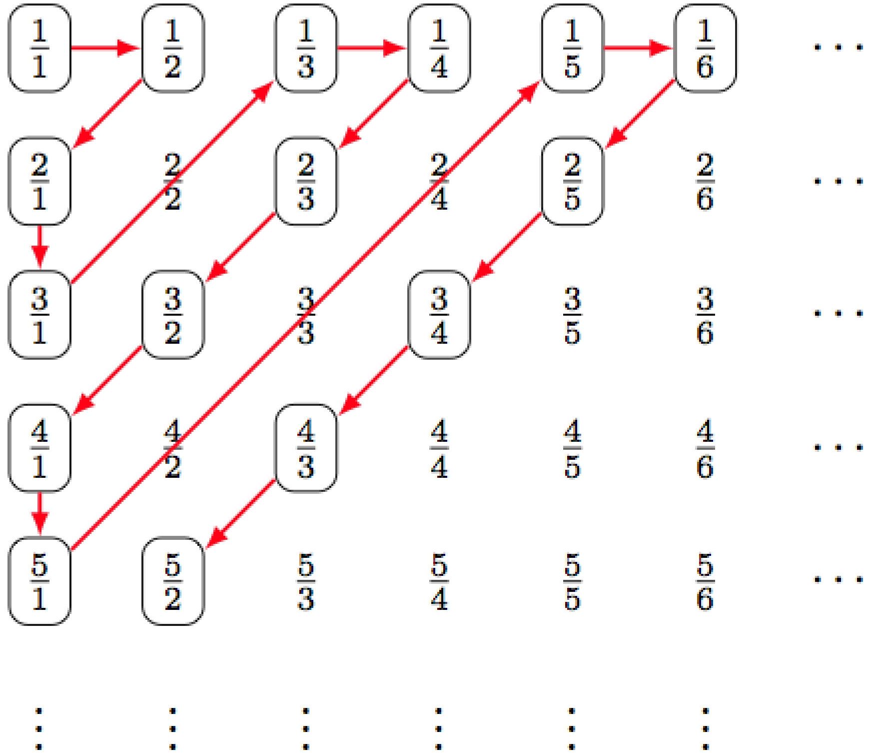
Countable Sets

- ✓ The set of Kleene star (Σ*) is **Countable**.
- ✓ Σ={a, b}   Σ* = {ε, a, b, aa, ab, ba, bb, aaa, ...}



Countable Sets

✓ The set of rational numbers is **Countable**.



f(N)→f(Q)

f(0)→f(1/1)

f(1)→f(1/2)

f(2)→f(2/1)

f(3)→f(3/1)

f(4)→f(1/3)

f(5)→f(1/4)

. . .

Countable Sets

- ✓ What about the set of **real numbers**?
- ✓ Is it a **countable** set?
- ✓ Did you hear Cantor's diagonal argument?

Countable Sets

- ✓ Cantor's diagonal argument about **real numbers**

Natural	Real
0	0.236436775676...
1	0.098473294543...
2	0.193214042202...
3	0.843279242093...
4	0.012934812343...
5	0.639423412934...
6	0.017773923845...
7	0.238920090909...
8	0.123984732999...
9	0.646329878122...
10	0.000123943437...
11	0.981298312892...
⋮	⋮

In this list, we wrote all numbers between [0 1]. Infinite numbers with each line of infinite length.

Countable Sets

✓ Cantor's diagonal argument on **real numbers**

Natural	Real
0	0.236436775676...
1	0.098473294543...
2	0.193214042202...
3	0.843279242093...
4	0.012934812343...
5	0.639423412934...
6	0.017773923845...
7	0.238920090909...
8	0.123984732999...
9	0.646329878122...
10	0.000123943437...
11	0.981298312892...
⋮	⋮
	0.293233992132... → ???

Can we find **a new number?**

Let's select the symbols on the diagonal in red.

Countable Sets

- ✓ Cantor's diagonal argument on **real numbers**

Natural	Real
0	0.236436775676...
1	0.098473294543...
2	0.193214042202...
3	0.843279242093...
4	0.012934812343...
5	0.639423412934...
6	0.017773923845...
7	0.238920090909...
8	0.123984732999...
9	0.646329878122...
10	0.000123943437...
11	0.981298312892...
⋮	⋮
	0.293233992132...

Can we find **a new number?**

Let's select the symbols on the diagonal in red.

0.293233992132... → it is probably somewhere up

Countable Sets

✓ Cantor's diagonal argument on **real numbers**

Natural	Real
0	0.236436775676...
1	0.098473294543...
2	0.193214042202...
3	0.843279242093...
4	0.012934812343...
5	0.639423412934...
6	0.017773923845...
7	0.238920090909...
8	0.123984732999...
9	0.646329878122...
10	0.000123943437...
11	0.981298312892...
⋮	⋮
	0.293233992132...
	0.746894310875...

If we change symbols in red we find a new number

Countable Sets

✓ If Cantor's diagonal argument is applied on the set of all languages over a finite alphabet such as binary symbols

s_1 : 0000000…	w_1 : aaaaaaa…
s_2 : 1111111…	w_2 : bbbbbbb…
s_3 : 0101010…	w_3 : abababa…
s_4 : 1010101…	w_4 : bababab…
s_5 : 1100110…	w_5 : aabaaba…
s_6 : 0011001…	w_6 : bbbabbb…
s_7 : 1001001…	w_7 : aabaaba…
s_d: 0100101…	w_d : abaaaba…

Countable vs. Enumerable

If P is Enumerable, then P is Countable.

- $(P \in E) \rightarrow (P \in C)$

But not vice versa. Σ^* is countable, so all its subsets are countable.

If every countable language were enumerable, then every language would be decidable.

- That’s all.
- Thanks for listening.