	0	1
∅	∅	∅
A	{B,C}	A
B	∅	{B,C}
C	C	C
{A,B}	{B,C}	{A,B}
{B,C}	C	{B,C}
{A,C}	{B,C}	{A,C}
{A,B,C}	{B,C}	{A,B,C}

Theory of Computation

Lesson 3

Converting a NFA to a DFA

Equivalence of DFAs and NFAs

- You may have the impression that nondeterministic finite automata are more powerful than deterministic finite automata.
- In this section, we will show that this is not the case. That is, we should know that a language can be accepted by a DFA if and only if it can be accepted by an NFA.
- What about converting a DFA to an NFA? Well, there is nothing to do, because a DFA is also an NFA.

Converting a NFA to a DFA

Theorem 2.5.1 *Let $N = (Q, \Sigma, \delta, q, F)$ be a nondeterministic finite automaton. There exists a deterministic finite automaton M, such that $L(M) = L(N)$.*

Let the DFA M be defined as $M = (Q', \Sigma, \delta', q', F')$, where

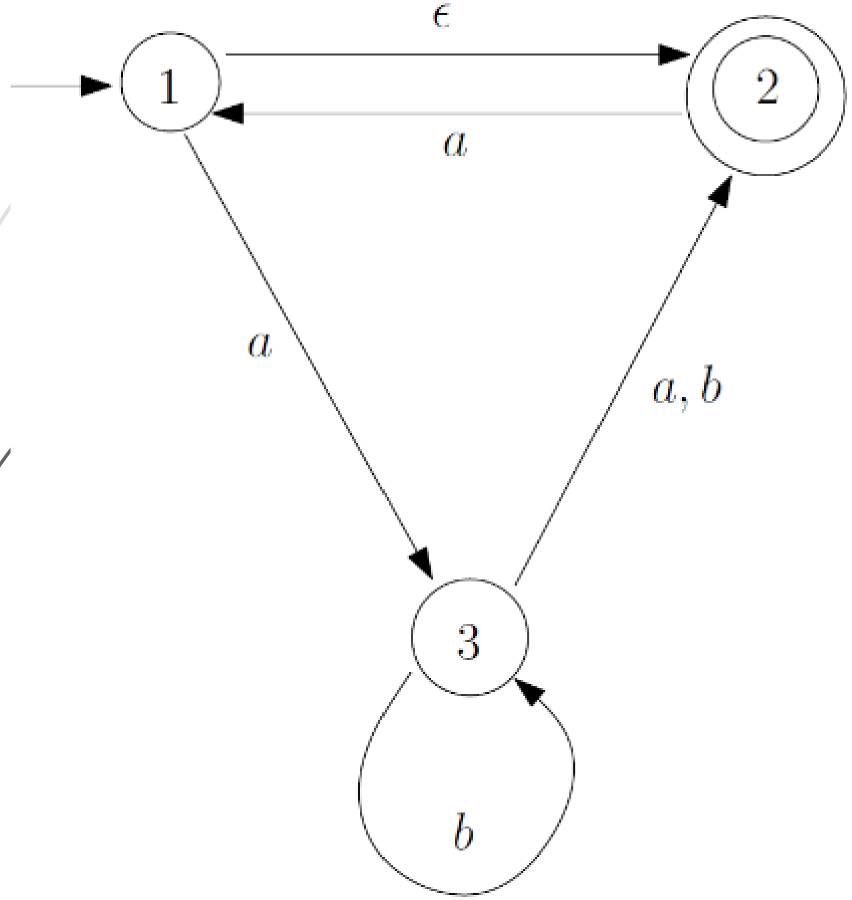
- the set Q' of states is equal to $Q' = \mathcal{P}(Q)$; observe that $|Q'| = 2^{|Q|}$,
- the start state q' is equal to $q' = \{q\}$; so M has the "same" start state as N,
- the set F' of accept states is equal to the set of all elements R of Q' having the property that R contains at least one accept state of N, i.e.,

$$F' = \{R \in Q' : R \cap F \neq \emptyset\},$$

- the transition function $\delta' : Q' \times \Sigma \to Q'$ is defined as follows: For each $R \in Q'$ and for each $a \in \Sigma$,

$$\delta'(R, a) = \bigcup_{r \in R} \delta(r, a).$$

NFA to DFA example



	a	b	ϵ
1	$\{3\}$	$\emptyset$	$\{2\}$
2	$\{1\}$	$\emptyset$	$\emptyset$
3	$\{2\}$	$\{2, 3\}$	$\emptyset$

NFA to DFA example

First, set of states Q' is easily determined by using power set of Q.

$$Q' = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}.$$

	a	b	ϵ
1	$\{3\}$	$\emptyset$	$\{2\}$
2	$\{1\}$	$\emptyset$	$\emptyset$
3	$\{2\}$	$\{2,3\}$	$\emptyset$

Writing the start state q' is also simple.

$$q' = C_\epsilon(\{q\}) = C_\epsilon(\{1\}) = \{1,2\}.$$

The third one, to find new accept states.

$$F' = \{\{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}\}.$$

NFA to DFA example

Now, it is time to define δ'

$$\delta'(\emptyset, a) = \emptyset \qquad \delta'(\emptyset, b) = \emptyset$$
$$\delta'(\{1\}, a) = \{3\} \qquad \delta'(\{1\}, b) = \emptyset$$
$$\delta'(\{2\}, a) = \{1,2\} \qquad \delta'(\{2\}, b) = \emptyset$$
$$\delta'(\{3\}, a) = \{2\} \qquad \delta'(\{3\}, b) = \{2,3\}$$
$$\delta'(\{1,2\}, a) = \{1,2,3\} \qquad \delta'(\{1,2\}, b) = \emptyset$$
$$\delta'(\{1,3\}, a) = \{2,3\} \qquad \delta'(\{1,3\}, b) = \{2,3\}$$
$$\delta'(\{2,3\}, a) = \{1,2\} \qquad \delta'(\{2,3\}, b) = \{2,3\}$$
$$\delta'(\{1,2,3\}, a) = \{1,2,3\} \qquad \delta'(\{1,2,3\}, b) = \{2,3\}$$

NFA to DFA example

- First, write all the elements of the Q' as the labels of rows and Σ as the label of columns.

	a	b	ϵ
1	$\{3\}$	$\emptyset$	$\{2\}$
2	$\{1\}$	$\emptyset$	$\emptyset$
3	$\{2\}$	$\{2,3\}$	$\emptyset$

	a	b
Ø		
{1}		
{2}		
{3}		
{1,2}		
{1,3}		
{2,3}		
{1,2,3}		

NFA to DFA example

The first row in the table should be filled with empty sets. It will be our black hole reject state.

	a	b	ϵ
1	$\{3\}$	$\emptyset$	$\{2\}$
2	$\{1\}$	$\emptyset$	$\emptyset$
3	$\{2\}$	$\{2,3\}$	$\emptyset$

	a	b
∅	∅	∅
{1}		
{2}		
{3}		
{1,2}		
{1,3}		
{2,3}		
{1,2,3}		

NFA to DFA example

Then import the partition matching the NFA into the table exactly.

	a	b	ϵ
1	$\{3\}$	$\emptyset$	$\{2\}$
2	$\{1\}$	$\emptyset$	$\emptyset$
3	$\{2\}$	$\{2,3\}$	$\emptyset$

	a	b
∅	∅	∅
{1}	{3}	∅
{2}	{1}	∅
{3}	{2}	{2,3}
{1,2}		
{1,3}		
{2,3}		
{1,2,3}		

NFA to DFA example

Now, use the ε column and update the section you just filled out.

	a	b	ϵ
1	$\{3\}$	$\emptyset$	$\{2\}$
2	$\{1\}$	$\emptyset$	$\emptyset$
3	$\{2\}$	$\{2,3\}$	$\emptyset$

	a	b
∅	∅	∅
{1}	{3}	∅
{2}	{1,2}	∅
{3}	{2}	{2,3}
{1,2}		
{1,3}		
{2,3}		
{1,2,3}		

NFA to DFA example

Last, fill the empty elements by using union operation among the filled elements.

	a	b	ϵ
1	$\{3\}$	$\emptyset$	$\{2\}$
2	$\{1\}$	$\emptyset$	$\emptyset$
3	$\{2\}$	$\{2,3\}$	$\emptyset$

	a	b
Ø	∅	∅
{1}	{3}	∅
{2}	{1,2}	∅
{3}	{2}	{2,3}
{1,2}	{1,2,3}	∅
{1,3}	{2,3}	{2,3}
{2,3}	{1,2}	{2,3}
{1,2,3}	{1,2,3}	{2,3}

NFA to DFA example

Now our DFA solution is ready. However, this may also include some unnecessary states. We can make an optimization so that we can delete unnecessary states.

q'={1,2}

	a	b
Ø	∅	∅
{1}	{3}	∅
{2}	{1,2}	∅
{3}	{2}	{2,3}
{1,2}	{1,2,3}	∅
{1,3}	{2,3}	{2,3}
{2,3}	{1,2}	{2,3}
{1,2,3}	{1,2,3}	{2,3}

NFA to DFA example

Finally, we found a DFA with only four states. Now we can draw its transition diagram.

q'={1,2}

F'=[~~{2},~~{1,2},{2,3},{1,2,3}]

	a	b
√ **Ø**	Ø	Ø
~~{1}~~	~~{3}~~	~~Ø~~
~~{2}~~	~~{1,2}~~	~~Ø~~
~~{3}~~	~~{2}~~	~~{2,3}~~
√ **{1,2}**	{1,2,3}	Ø
~~{1,3}~~	~~{2,3}~~	~~{2,3}~~
√ **{2,3}**	{1,2}	{2,3}
√ **{1,2,3}**	{1,2,3}	{2,3}

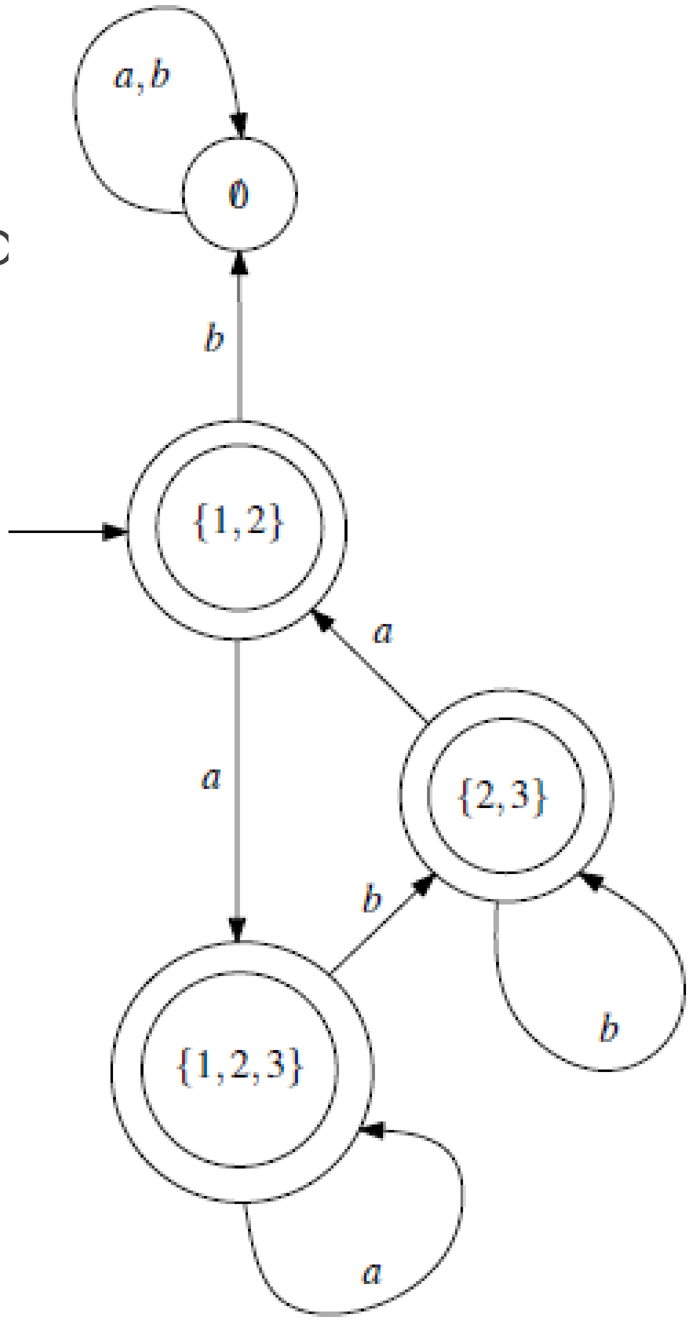
NFA to DFA example

Here, we have both the table and the transitic
diagram of the DFA solution.

q'={1,2}

F'=[{1,2},{2,3},{1,2,3}]

	a	b
Ø	∅	∅
{1,2}	{1,2,3}	∅
{2,3}	{1,2}	{2,3}
{1,2,3}	{1,2,3}	{2,3}



- That’s all.
- Thanks for listening.