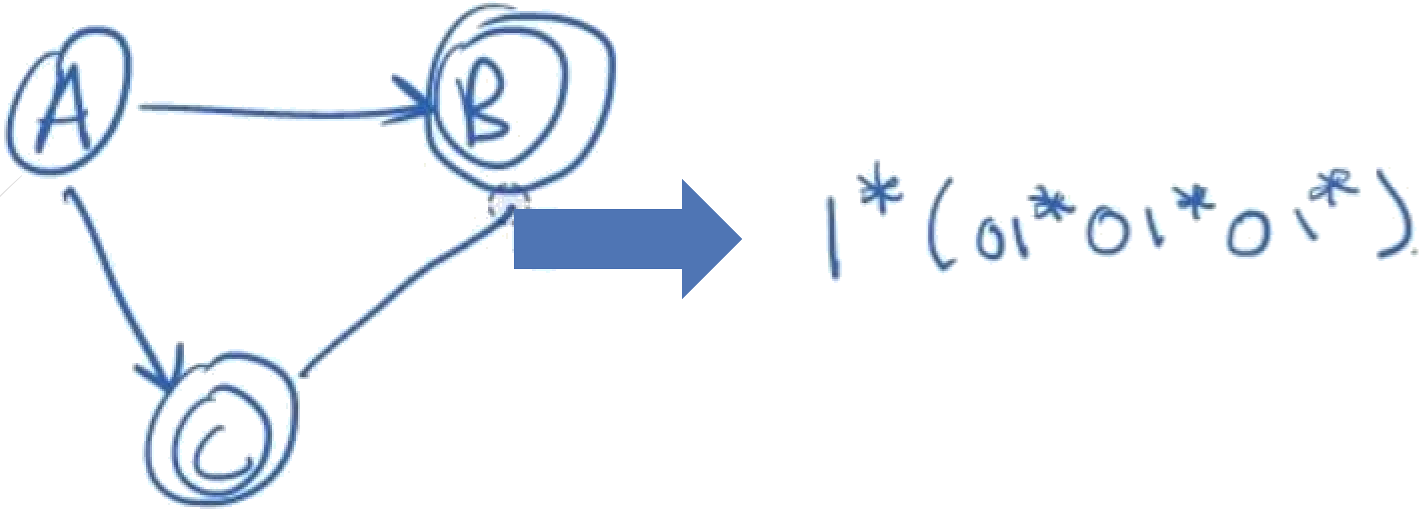


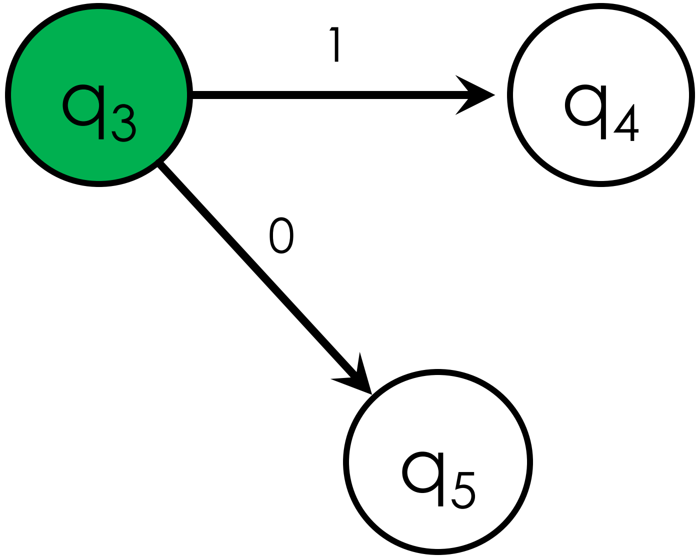
Theory of Computation

Lesson 6

Converting DFA to RegEx

DFA to RegEx

In this transformation approach, unlike the others, outgoing transitions of each state are modeled. Finally, the aim is to reach a reduced expression for the start state.

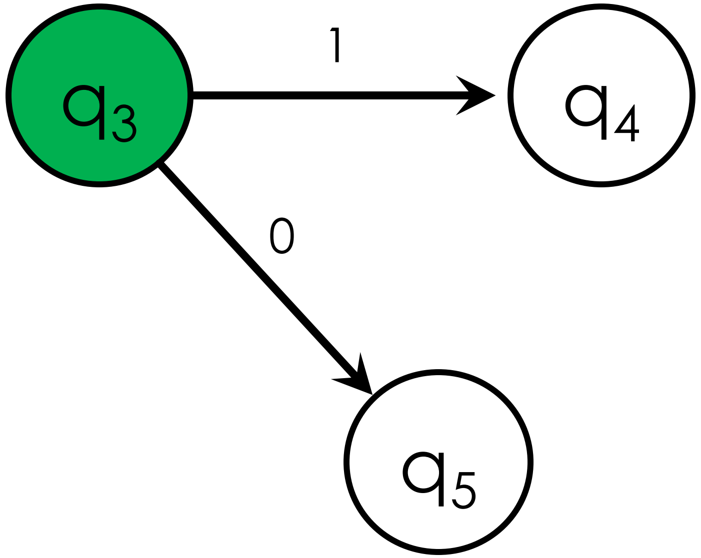


$$L_r = \bigcup_{a \in \Sigma} a \cdot L_{\delta(r,a)} \qquad \text{if } r \notin F$$

$$L_r = \epsilon \cup \left(\bigcup_{a \in \Sigma} a \cdot L_{\delta(r,a)} \right) \qquad \text{if } r \in F$$

DFA to RegEx

In the example below, let's just focus on the q_3 state. When modeling the state for RegEx, an expression is written for each outgoing as "**Symbol** concatenation **Target State**".

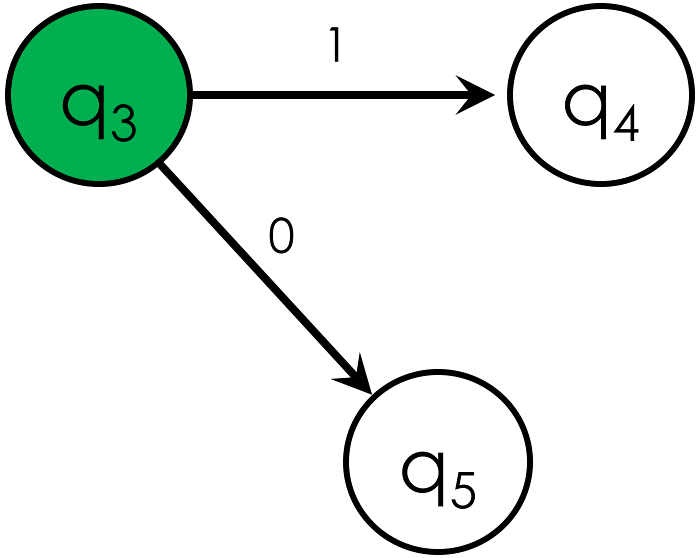


$$L_r = \bigcup_{a \in \Sigma} a \cdot L_{\delta(r,a)} \qquad \text{if } r \notin F$$

$$L_r = \epsilon \cup \left(\bigcup_{a \in \Sigma} a \cdot L_{\delta(r,a)} \right) \qquad \text{if } r \in F$$

DFA to RegEx

Since state q_3 is not an accept state, we will use the first rule.



$$L_r = \bigcup_{a \in \Sigma} a \cdot L_{\delta(r,a)} \qquad \text{if } r \notin F$$

$$L_{q_3} = 1 \cdot L_{q_4} \cup 0 \cdot L_{q_5}$$

DFA to RegEx

But there is a problem here. If we continue with this approach, it will create a lot of recursion. That's why we need a solution for recursive expressions.

$$L = B \cdot L \cup C$$

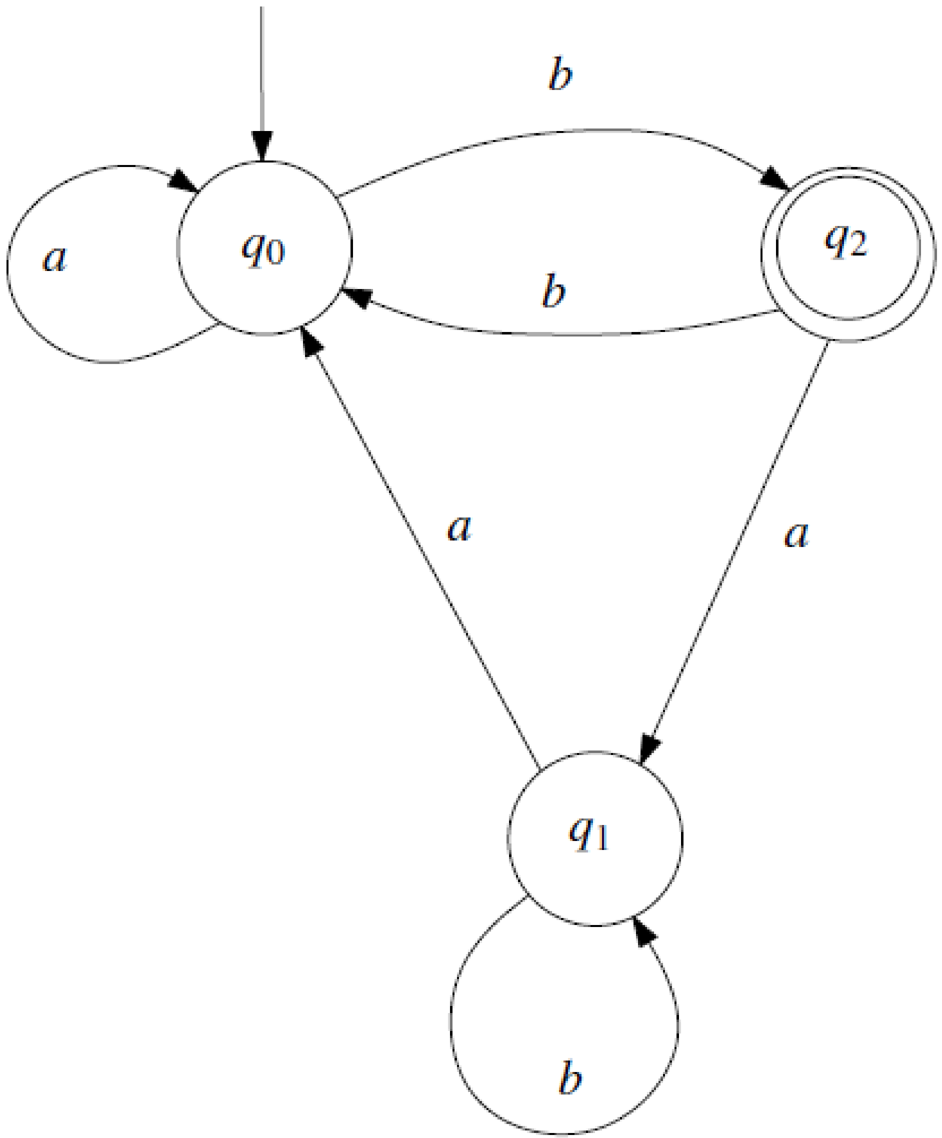
⬇

$$L = B^* C$$

A first example

Let's transform the DFA transition diagram on the right into a regular expression.

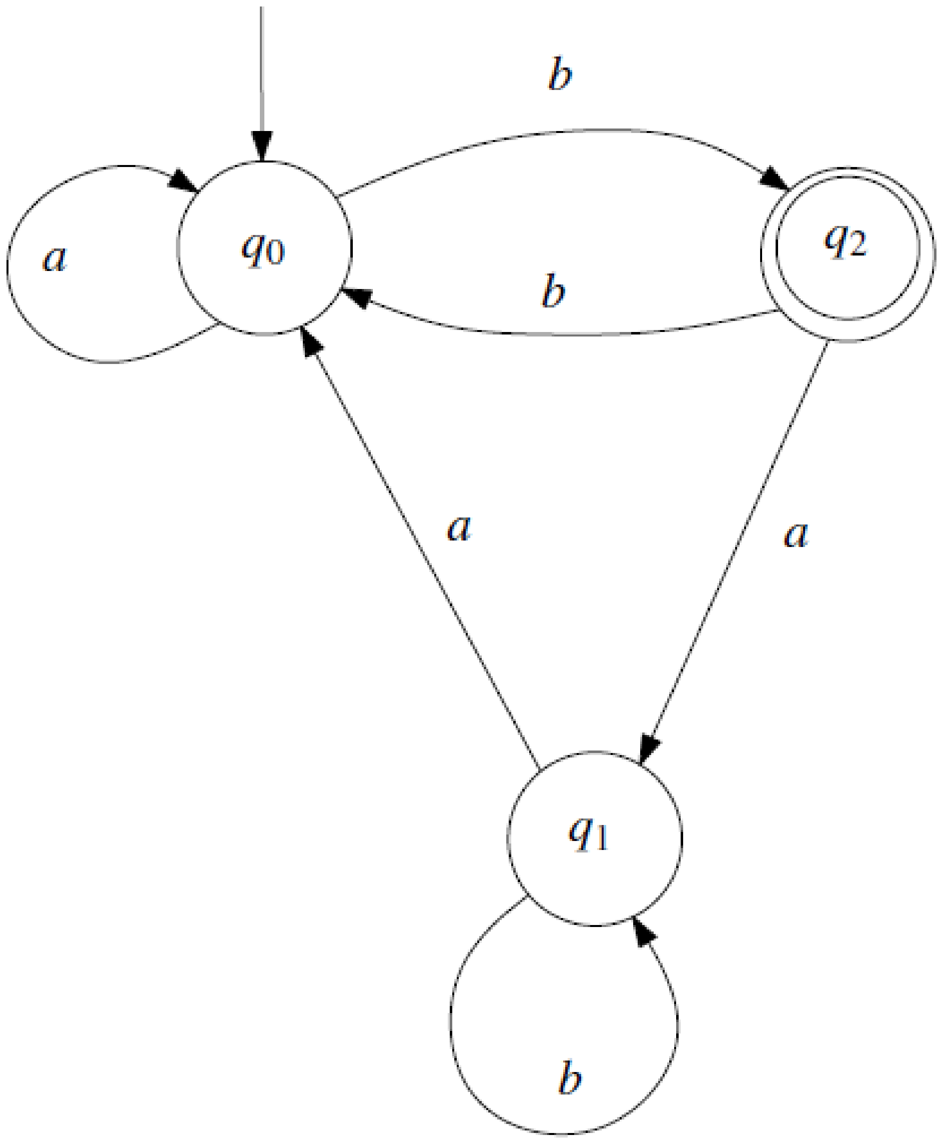
We must first start by modeling the expression of each state.



A first example

Our final goal is to write the reduced expression of the start state.

$$
\begin{aligned}
L_{q0} &= a \cdot L_{q0} \cup b \cdot L_{q2} \\
L_{q1} &= a \cdot L_{q0} \cup b \cdot L_{q1} \\
L_{q2} &= \epsilon \cup a \cdot L_{q1} \cup b \cdot L_{q0}
\end{aligned}
$$



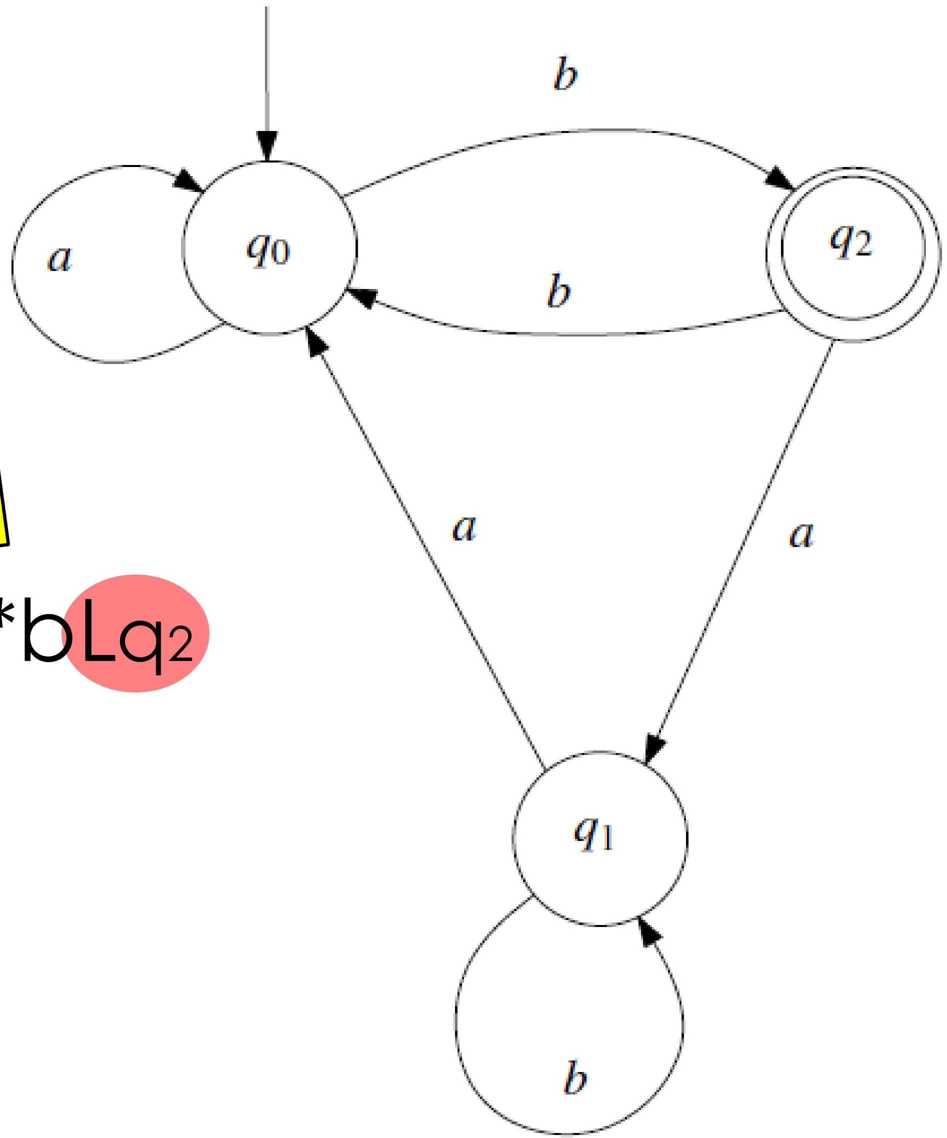
A first example

When we use our solution to clean up the recursions, these two expressions will turn into:

$L = B \cdot L \cup C$

$$
\begin{aligned}
L_{q0} &= a \cdot L_{q0} \cup b \cdot L_{q2} \\
L_{q1} &= a \cdot L_{q0} \cup b \cdot L_{q1} \\
L_{q2} &= \epsilon \cup a \cdot L_{q1} \cup b \cdot L_{q0}
\end{aligned}
$$

→ a^*bL_{q2}

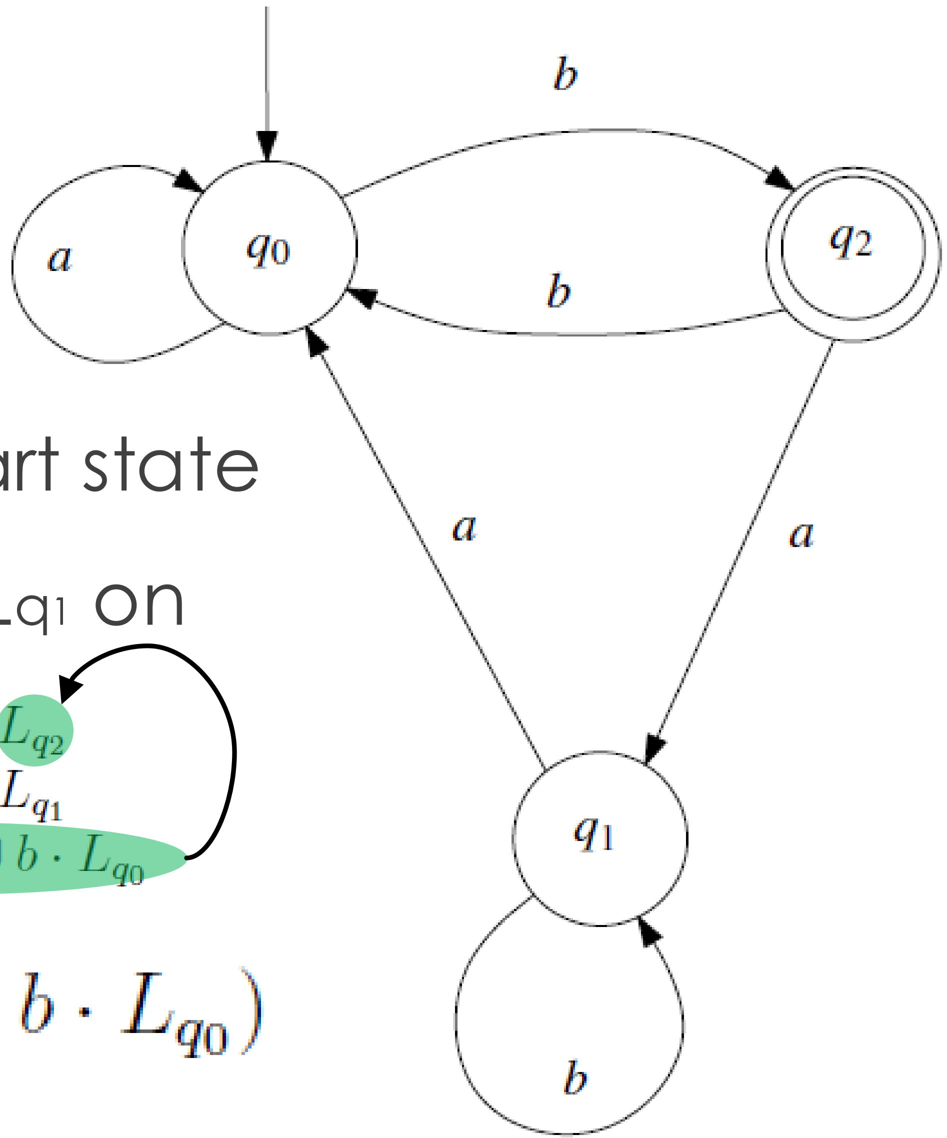


A first example

When we write the expression of the q_2 state instead of the variable L_{q_2} on the right hand side of the start state expression, we see the expression L_{q_1} on the right this time.

$$
\begin{aligned}
L_{q_0} &= a \cdot L_{q_0} \cup b \cdot L_{q_2} \\
L_{q_1} &= a \cdot L_{q_0} \cup b \cdot L_{q_1} \\
L_{q_2} &= \epsilon \cup a \cdot L_{q_1} \cup b \cdot L_{q_0}
\end{aligned}
$$

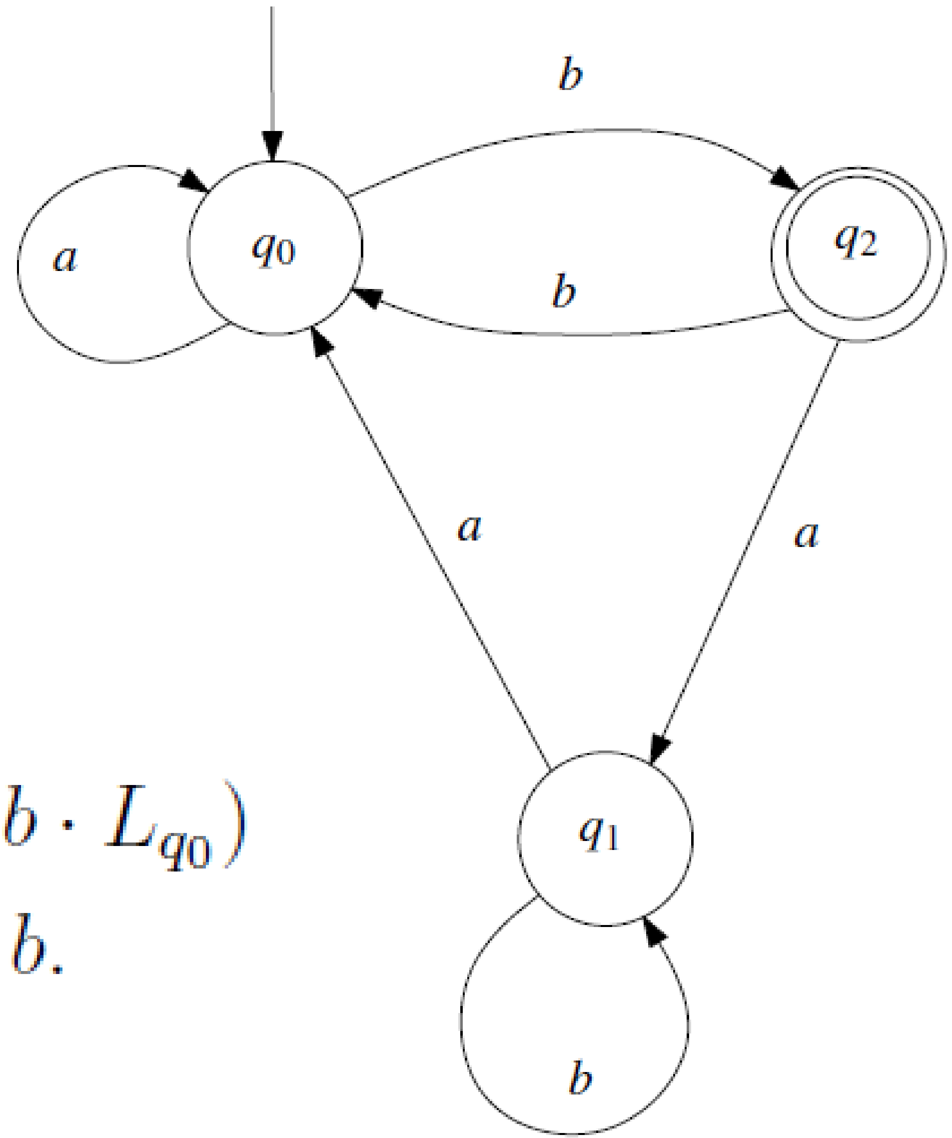
$$L_{q_0} = a \cdot L_{q_0} \cup b \cdot (\epsilon \cup a \cdot L_{q_1} \cup b \cdot L_{q_0})$$



A first example

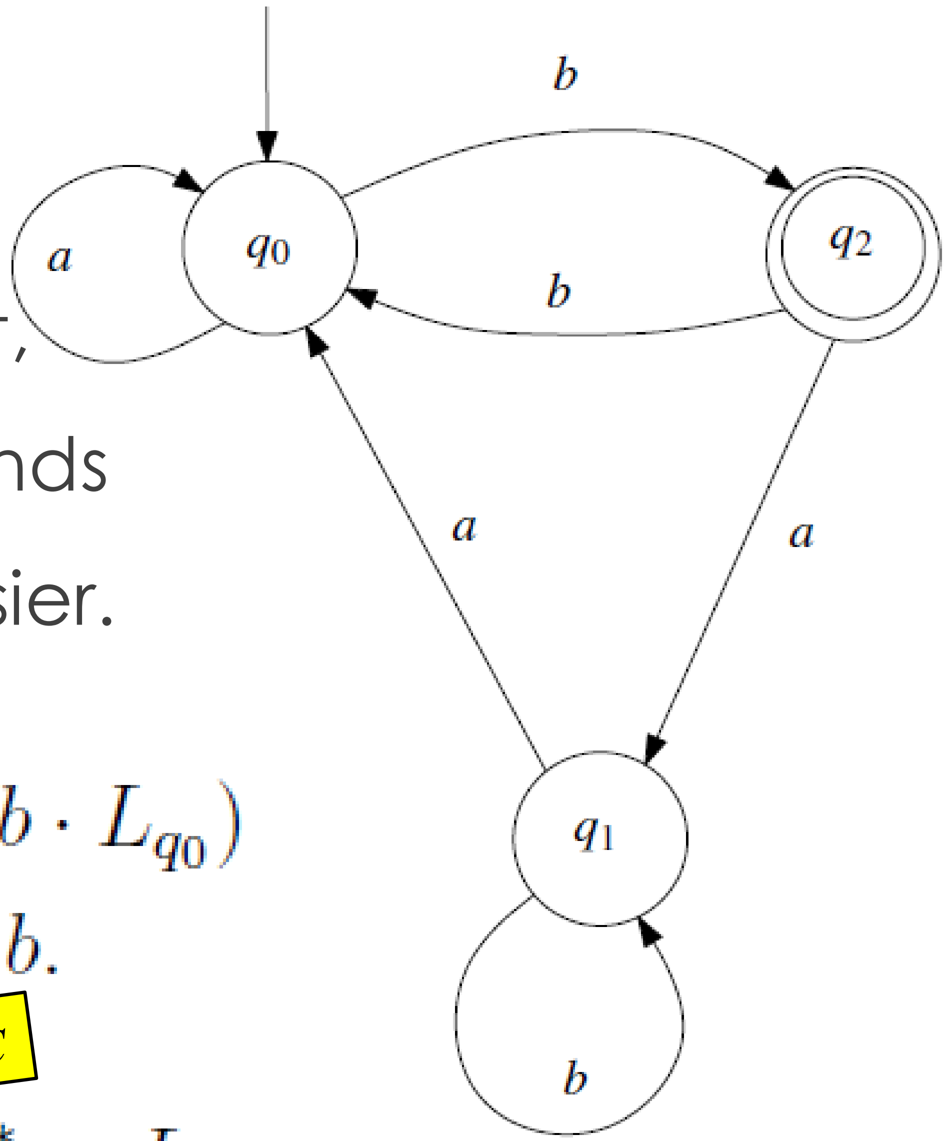
After arranging the equation we have in common brackets, we will understand that we should get rid of the unwanted variable L_{q1}.

$$\begin{aligned} L_{q0} &= a \cdot L_{q_0} \cup b \cdot (\epsilon \cup a \cdot L_{q_1} \cup b \cdot L_{q_0}) \\ &= (a \cup bb) \cdot L_{q0} \cup ba \cdot L_{q_1} \cup b. \end{aligned}$$



A first example

Fortunately, the L_{q1} expression fits our recursive solution. Not only that, but the obtained expression depends only on L_{q0}. This made our work easier.



$$\begin{aligned} L_{q0} &= a \cdot L_{q0} \cup b \cdot (\epsilon \cup a \cdot L_{q1} \cup b \cdot L_{q0}) \\ &= (a \cup bb) \cdot L_{q0} \cup ba \cdot L_{q1} \cup b. \end{aligned}$$

$L = B \cdot L \cup C$

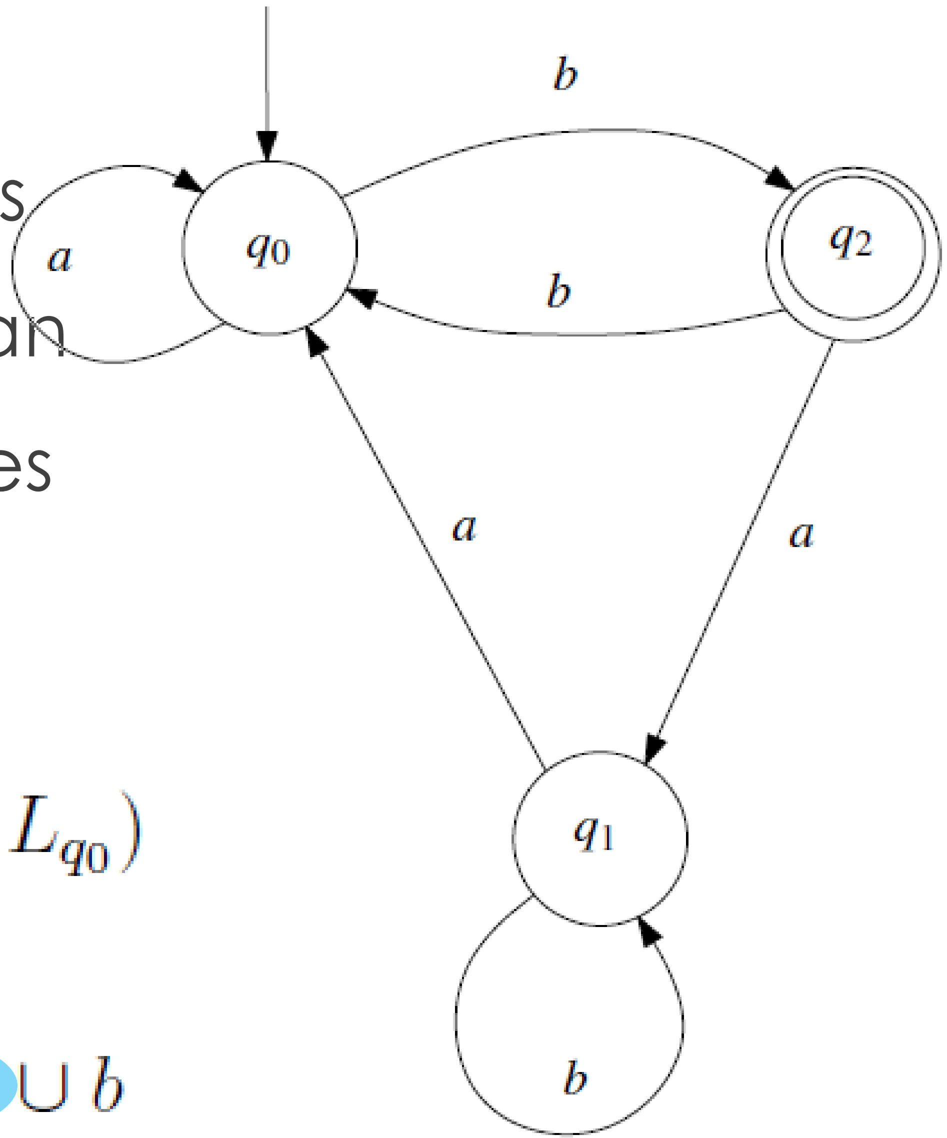
$$L_{q1} = a \cdot L_{q0} \cup b \cdot L_{q1} \Rightarrow b^* a \cdot L_{q0}$$

A first example

The simplified expression we have is dependent only on L_{q_0}. Now we can get rid of all our unwanted variables by doing the recursive solution.

$$
\begin{aligned}
L_{q_0} &= a \cdot L_{q_0} \cup b \cdot (\epsilon \cup a \cdot L_{q_1} \cup b \cdot L_{q_0}) \\
&= (a \cup bb) \cdot L_{q_0} \cup ba \cdot L_{q_1} \cup b.
\end{aligned}
$$

$$
L_{q_0} = (a \cup bb) \cdot L_{q_0} \cup ba \cdot b^* a \cdot L_{q_0} \cup b
$$



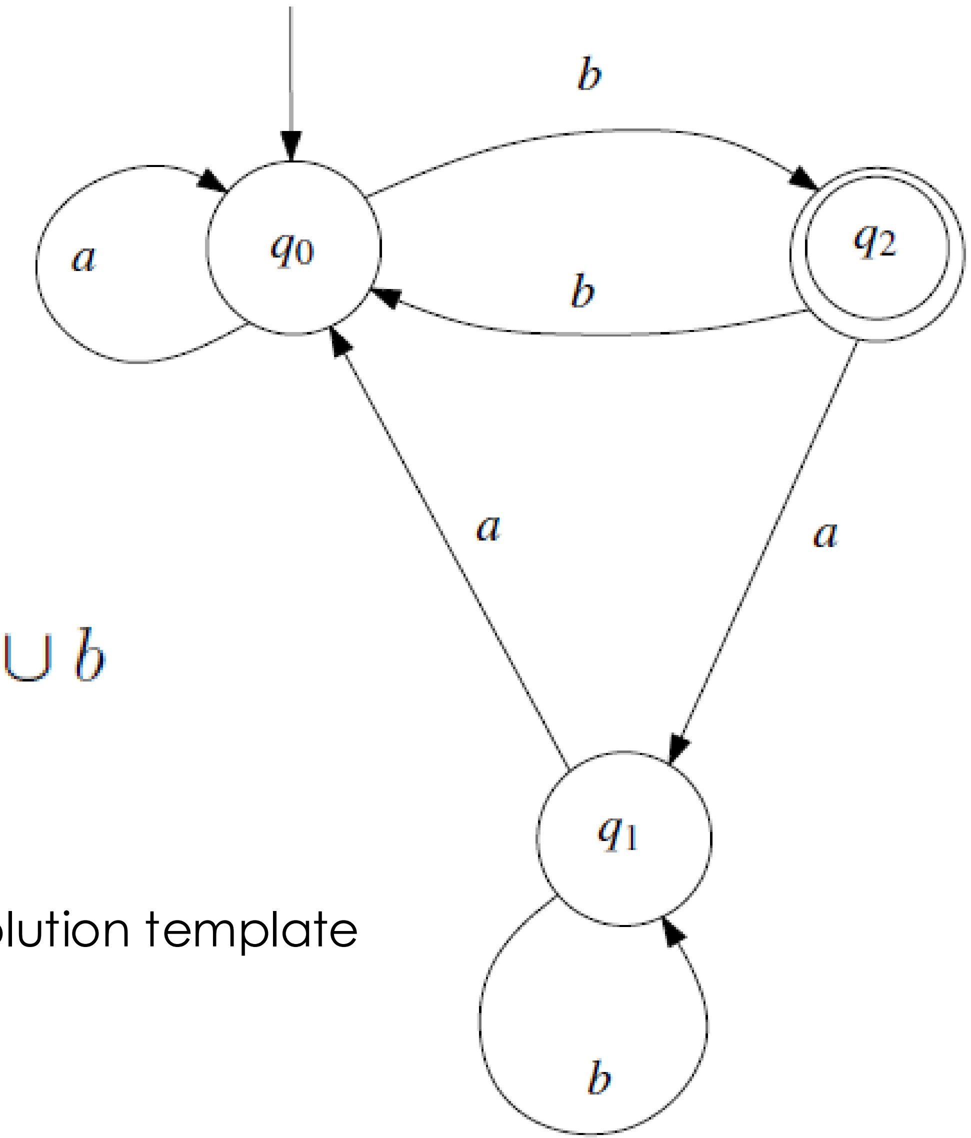
A first example

Finally, we obtained the regular expression of the DFA transition diagram on the right.

$$L_{q_0} = (a \cup bb) \cdot L_{q_0} \cup ba \cdot b^* a \cdot L_{q_0} \cup b$$
$$= (a \cup bb \cup bab^* a) L_{q_0} \cup b.$$

$L = B \cdot L \cup C$ Remember our solution template

$$(a \cup bb \cup bab^* a)^* b$$



- That’s all.
- Thanks for listening.