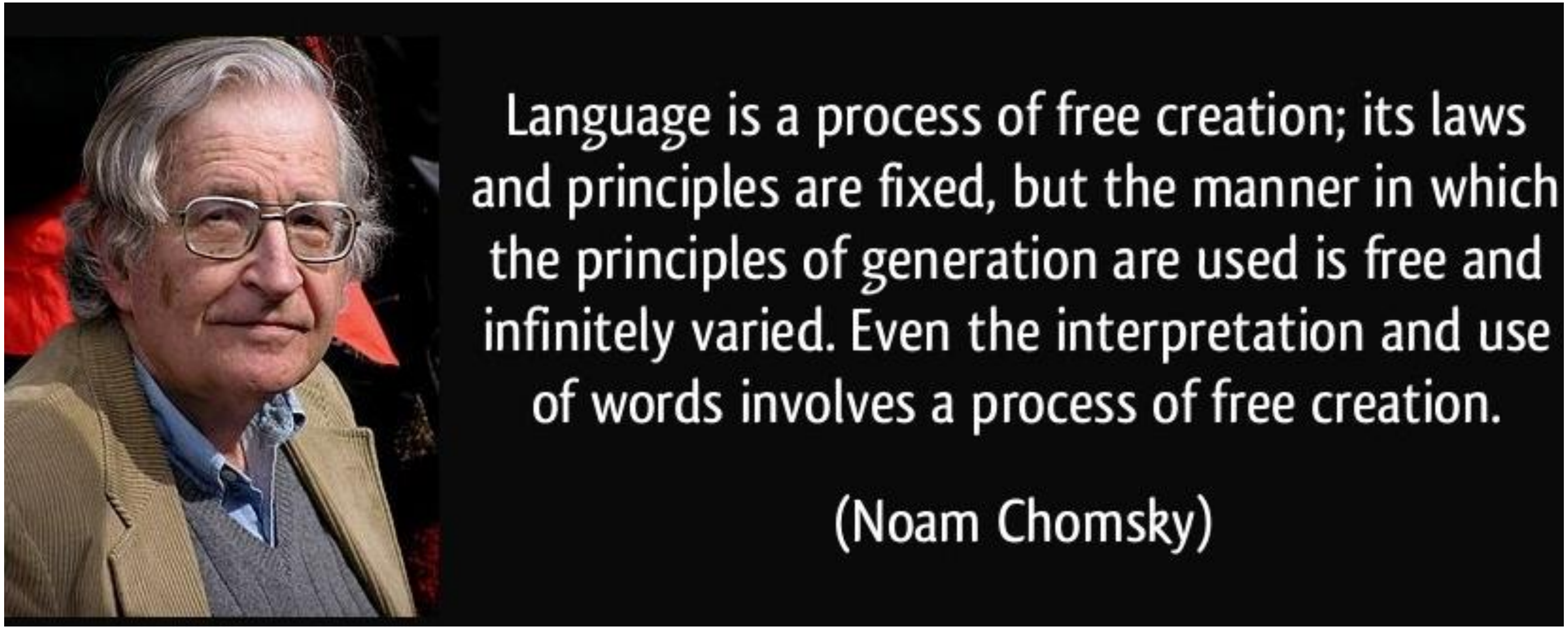
$$S \rightarrow 0\,S\,1 \mid \varepsilon$$

Theory of Computation

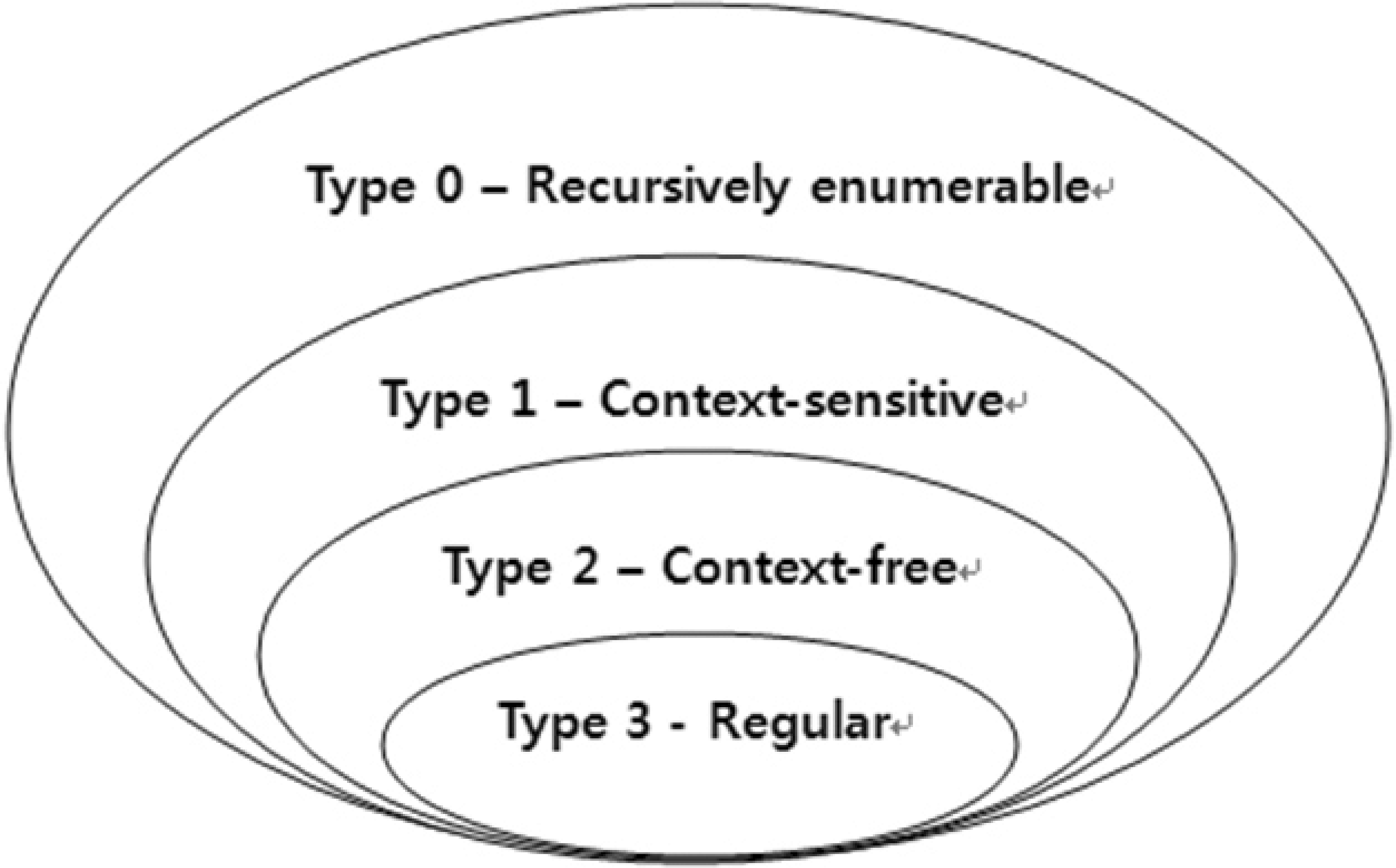
Lesson 7

Context Free Languages

Noam Chomsky



Chomsky Hierarchy



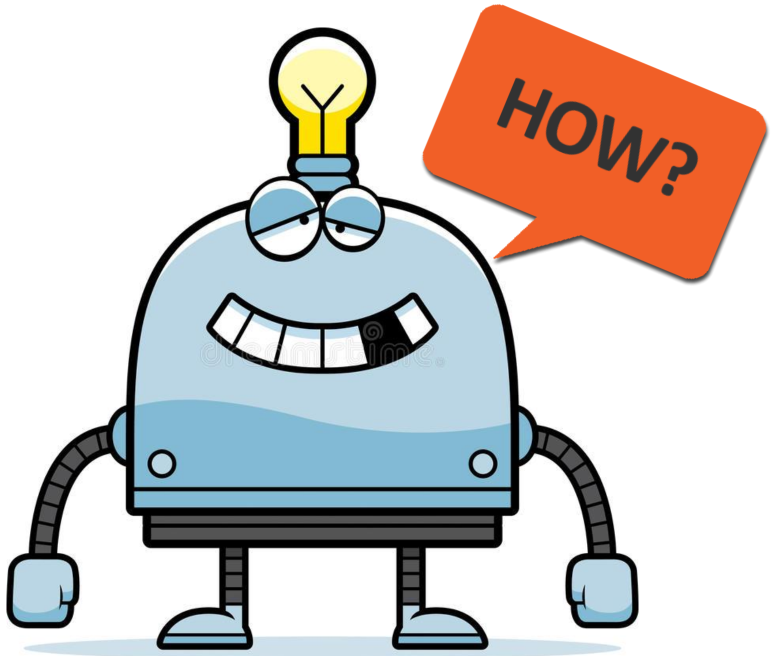
Context Free Grammar

S --> NP VP	1.0	NP --> NP PP	0.4
PP --> P NP	1.0	NP --> *astronomers*	0.1
VP --> V NP	0.7	NP --> *ears*	0.18
VP --> VP PP	0.3	NP --> *saw*	0.04
P --> *with*	1.0	NP --> *stars*	0.18
V --> *saw*	1.0	NP --> *telescopes*	0.1

P(*astronomers saw stars with ears*) = ?

Context Free Grammar

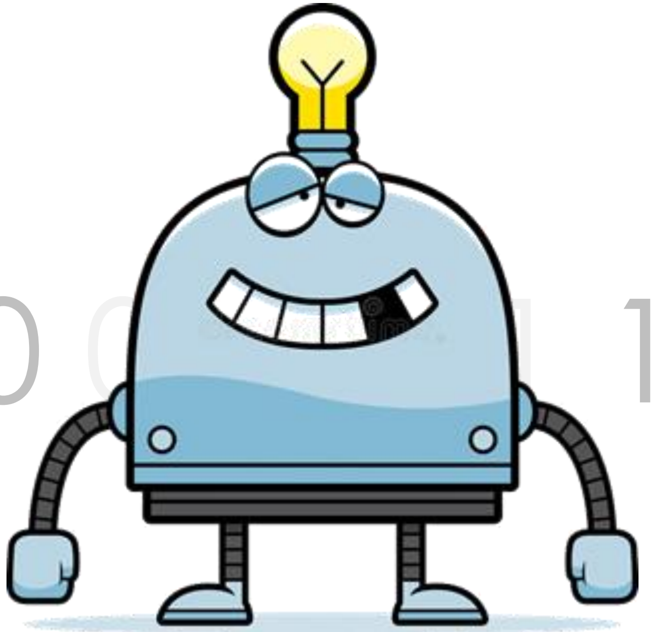
How can we generate a 0^n1^n word or sentence with memoryless tools just like in regular language?



Context Free Grammar

This is also a method that people with a weak memory can use: **getting the job done on time**.

0 0 0 0

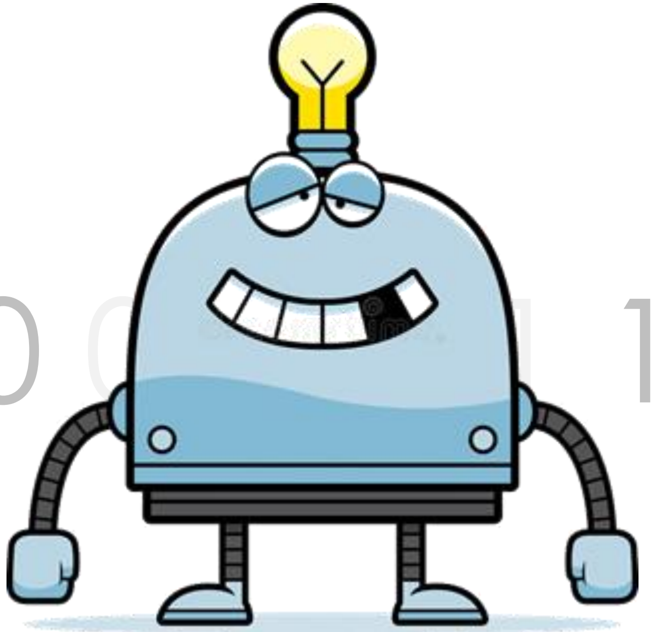


1 1 1 1

Context Free Grammar

$$S \rightarrow 0\,S\,1$$

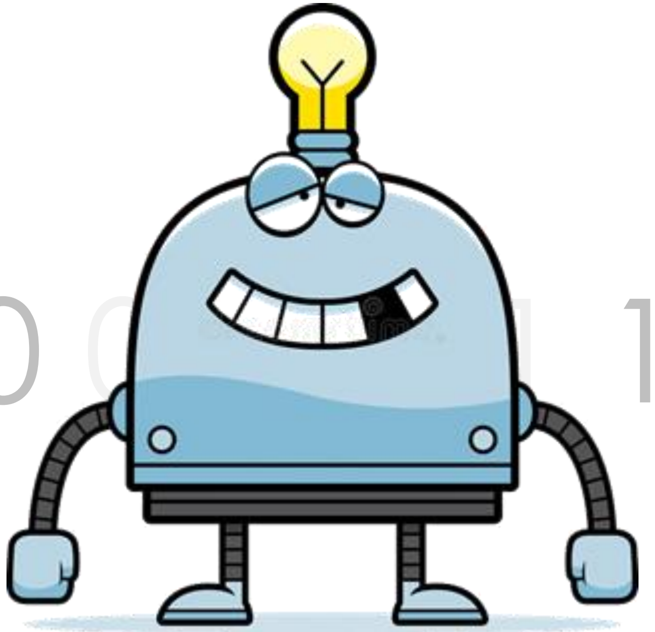
0 0 0 0 0 1 1 1 1 1



Context Free Grammar

$$S \rightarrow 0\,S\,1 \mid \varepsilon$$

OR



0 0 0 0 0 1 1 1 1 1

Formal Definition

A context-free grammar is a 4-tuple $G = (V,\Sigma,R,S)$ where

- V is a finite set, whose elements are called variables,
- Σ is a finite set, whose elements are called terminals,
- $V \cap \Sigma = \emptyset$,
- S is an element of V; it is called the start variable,
- R is a finite set, whose elements are called rules. Each rule has the form $A \rightarrow w$, where A is element of V and w is element of $(V \cup \Sigma)^*$.

Derivation of ‘aaaabb’

a^n b^m

$$S \xrightarrow{1} AB$$
$$A \xrightarrow{2} a$$
$$A \xrightarrow{3} aA$$
$$B \xrightarrow{4} b$$
$$B \xrightarrow{5} bB$$

$$S \overset{1}{\Rightarrow} AB$$

Derivation of ‘aaaabb’

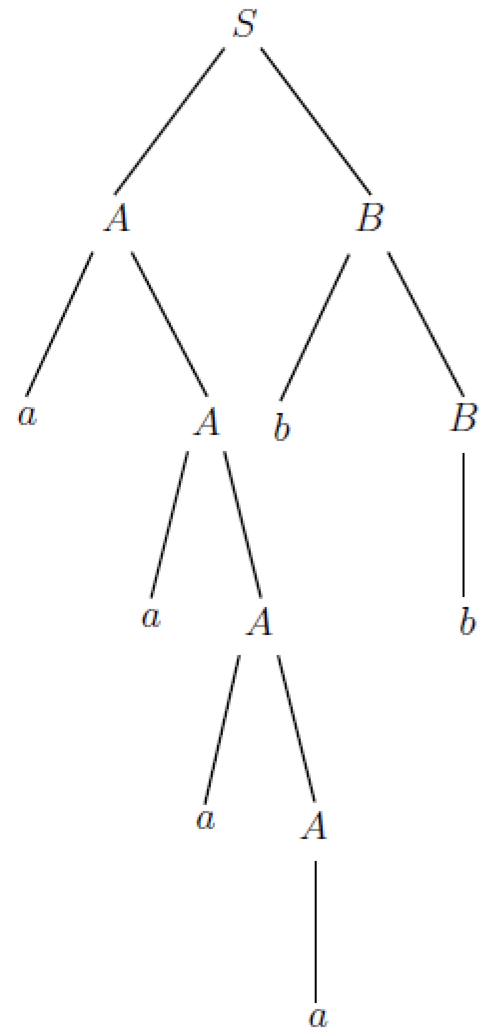
$$
\begin{aligned}
S &\xrightarrow{1} AB \\
A &\xrightarrow{2} a \\
A &\xrightarrow{3} aA \\
B &\xrightarrow{4} b \\
B &\xrightarrow{5} bB
\end{aligned}
$$

$$
\begin{aligned}
S &\overset{1}{\Rightarrow} AB \\
&\overset{3}{\Rightarrow} aAB
\end{aligned}
$$

Derivation of ‘aaaabb’

$$S \xrightarrow{1} AB$$
$$A \xrightarrow{2} a$$
$$A \xrightarrow{3} aA$$
$$B \xrightarrow{4} b$$
$$B \xrightarrow{5} bB$$

$$\begin{aligned} S &\overset{1}{\Rightarrow} AB \\ &\overset{3}{\Rightarrow} aAB \\ &\overset{5}{\Rightarrow} aAbB \\ &\overset{3}{\Rightarrow} aaAbB \\ &\overset{3}{\Rightarrow} aaaAbB \\ &\overset{2}{\Rightarrow} aaaabB \\ &\overset{4}{\Rightarrow} aaaabb \end{aligned}$$



New Grammar

How can we derivate ‘1010101’

Improve the grammar

$S \rightarrow 1S1 \mid 0S0 \mid \varepsilon$ 101 ε 101

$S \rightarrow 1S1 \mid 0S0 \mid 0 \mid 1$ 101 0 101

101 1 101

$S \rightarrow 1S1 \mid 0S0 \mid 0 \mid 1 \mid \varepsilon$

$S \rightarrow 10S \mid 01S \mid 0S1 \mid 1S0 \mid \varepsilon$

A first example: Properly nested parentheses

In this first example, we will model the spelling of parenthetical symbols used in mathematics. You know, when a left parenthesis is used, it has to match a right parenthesis typed later. Otherwise, there will be a typo in the use of parentheses.

$(3*(x^{(x-2)}+1))-(y+1)*(x+5)$

((( ))) ( ) ( )

aaabbb ab ab

A first example: Properly nested parentheses

There are three critical details that need to be modeled here:

((( ))) ( ) ( )

aaabbb ab ab

A first example: Properly nested parentheses

There are three critical details that need to be modeled here:

1. Nested parentheses,

((( ))) ( ) ( )

aaabbb ab ab

$S \rightarrow aSb$

A first example: Properly nested parentheses

There are three critical details that need to be modeled here:

1. Nested parentheses,
2. Side-by-side parentheses,

((( ))) ( ) ( )

aaabbb ab ab

$S \rightarrow aSb$ $S \rightarrow SS$

A first example: Properly nested parentheses

There are three critical details that need to be modeled here:

1. Nested parentheses,
2. Side-by-side parentheses,
3. Termination of recursive steps

((( ))) ( ) ( )

aaabbb ab ab

$S \rightarrow aSb$ $S \rightarrow SS$

$S \rightarrow \varepsilon$

A second example: $a^n b^m c^{n+m}$

$a^3 b^2 c^5$

Consider the language

$L=\{a^n b^m c^{n+m} : n \geq 0, m \geq 0\}$

A second example: $a^n b^m c^{n+m}$

Consider the language

$L=\{a^n b^m c^{n+m} : n \geq 0, m \geq 0\}$

$a^3 b^2 c^5$

aaa bb ccccc

$S \rightarrow aSc$ $S \rightarrow bSc$

A second example: $a^n b^m c^{n+m}$

Consider the language

$L=\{a^n b^m c^{n+m} : n \geq 0, m \geq 0\}$

$a^3b^2c^5$

aaa bb ccccc

$S \to aSc$ $S \to bSc$

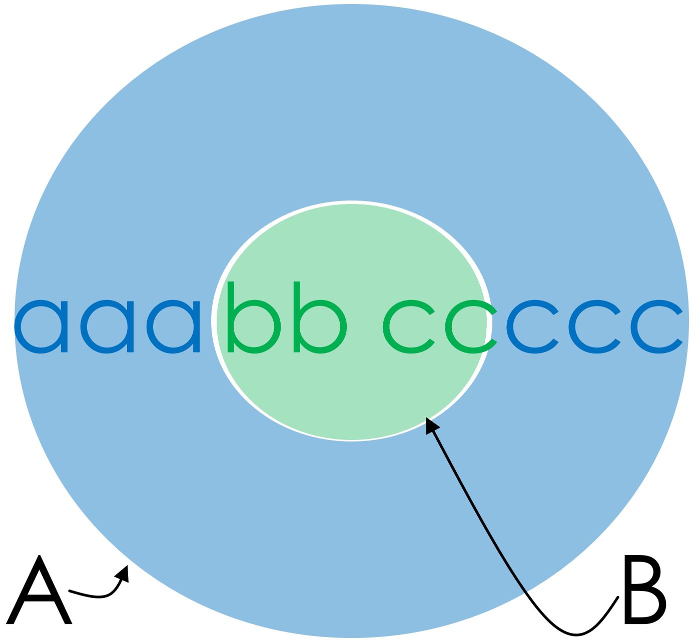
🚫 abaccc

A second example: $a^n b^m c^{n+m}$

Consider the language

$L=\{a^n b^m c^{n+m} : n \geq 0, m \geq 0\}$

$S \rightarrow A \mid \varepsilon$



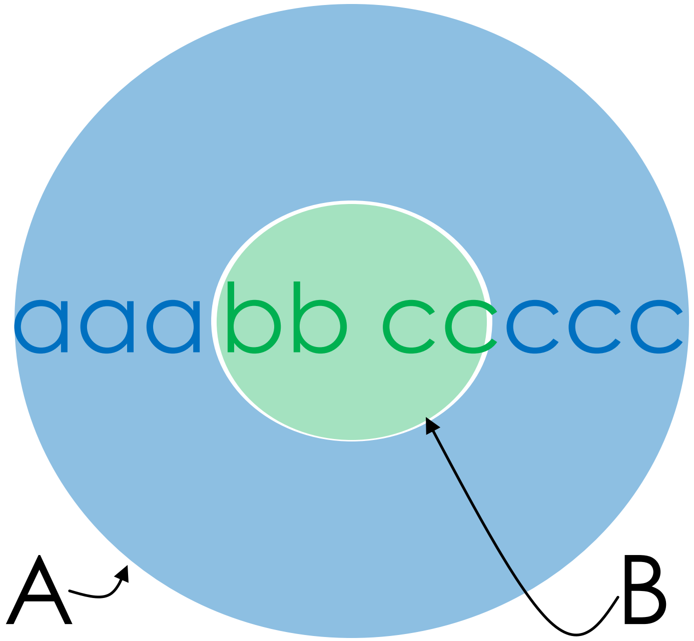
A second example: $a^n b^m c^{n+m}$

Consider the language

$L = \{a^n b^m c^{n+m} : n \geq 0, m \geq 0\}$

$S \rightarrow A \mid \varepsilon$

$A \rightarrow aAc \mid B \mid \varepsilon$



A second example: $a^n b^m c^{n+m}$

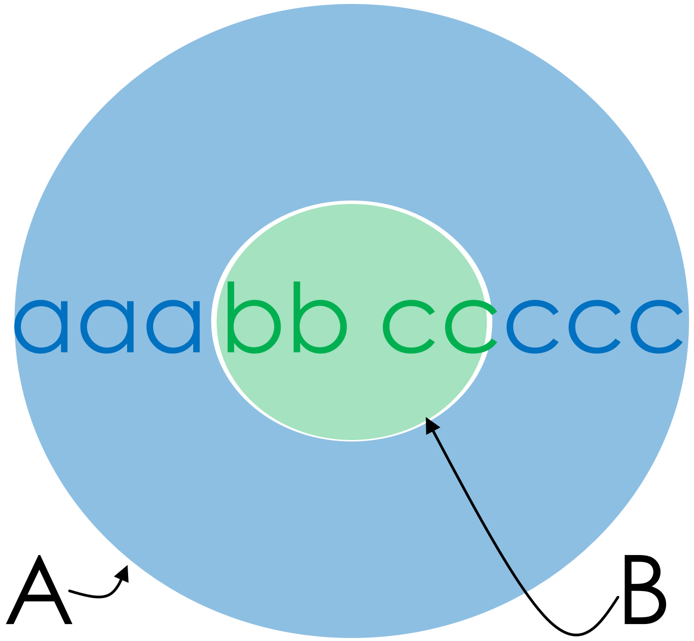
Consider the language

$L = \{a^n b^m c^{n+m} : n \geq 0, m \geq 0\}$

$S \rightarrow A \mid \varepsilon$

$A \rightarrow aAc \mid B \mid \varepsilon$

$B \rightarrow bBc \mid \varepsilon$



- That’s all.
- Thanks for listening.